PDF Method for Power Systems

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1 Introduction

In Fig. 1, mechanical power $P_\mathrm{m}(t)$ is converted through a generator, which rotates at angular speed ω, to an electrical output of power P_e and voltage E. The generator's transient reactance is X and its electrical angle is θ. The electrical power is utilized to drive the load whose voltage is denoted by V.

The basic equation for generator stability analysis can be derived from the Newton second law and the phasor diagram for electrical power output

$$\frac{2H}{\omega_s}\frac{\mathrm{d}\omega}{\mathrm{d}t} = P_\mathrm{m} - P_\mathrm{e} \tag{1}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega - \omega_s, \tag{2}$$

$$P_\mathrm{e} = \frac{EV}{X}\sin(\theta - \theta_1), \tag{3}$$

where the generator's inertia is H and ω_s denotes the synchronization speed. When the system includes renewable energy, such as wind and solar power, the mechanical power input $P_\mathrm{m}(t)$ becomes a random parameter.

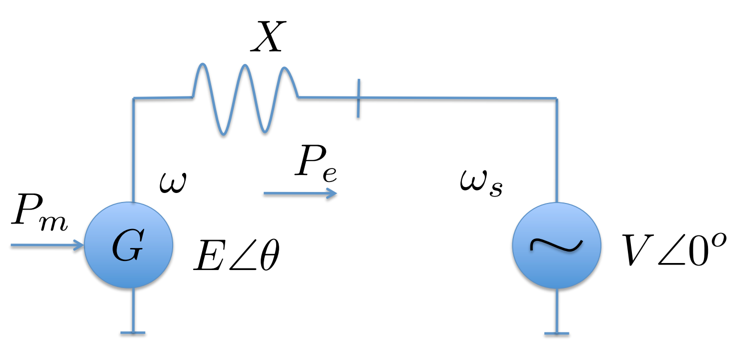

Figure 1: Diagram of power system.

2 PDF Method

In the current study, we first substitute Eq. (3) into Eq. (1) and obtain

$$\frac{2H}{\omega_s}\frac{\mathrm{d}\omega}{\mathrm{d}t} = P_{\mathrm{m}} - \frac{EV}{X}\sin\theta \tag{4}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega - \omega_s. \tag{5}$$

The above equations describe a dynamic system driven a colored noise $P_{\mathrm{m}}(t)$ with the mean $\langle P_m \rangle$.

Employing previous work on PDF method, we start by introducing a functional, in the form of dirac delta function that represents "raw" (or "fine-grained") probabilistic density function (PDF),

$$\Pi(\Theta, \Omega; t) = \delta[\Theta - \theta(t)]\,\delta[\Omega - \omega(t)], \tag{6}$$

where Θ and Ω are deterministic values (outcomes) that the random quantities θ and ω can take at time t, respectively. Let $p_{\theta,\omega}(\Theta, \Omega; t)$ denote the joint probability density function (PDF) of electrical angle and radial velocity at the time t. The ensemble average (over random θ and ω) of (6) yields the joint PDF,

$$\langle \Pi(\Theta, \Omega; t)\rangle \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta[\Theta - \theta']\,\delta[\Omega - \omega']\,p_{\theta,\omega}(\theta', \omega'; t)\,\mathrm{d}\theta'\mathrm{d}\omega' = p_{\theta,\omega}(\Theta, \Omega; t). \tag{7}$$

Multiplying the stochastic ODEs with the derivatives of Π, averaging in the probability space and using the "Large Eddy Diffusivity" closure approximation lead to the closed-form equation for the joint PDF $p_{\theta,\omega}(\Theta$:

$$\frac{\partial p_{\theta,\omega}}{\partial t} = -\frac{\partial}{\partial \Theta}\sqcap_1 p_{\theta,\omega} - \frac{\partial}{\partial \Omega}\sqcap_2 p_{\theta,\omega} + \frac{\partial}{\partial \Omega}\left(\mathcal{D}\frac{\partial p_{\theta,\omega}}{\partial \Omega}\right), \tag{8a}$$

$$\sqcap_1(\Omega, t) = \Omega - \omega_{\mathrm{s}}, \qquad \sqcap_2(\Theta, t) = -\frac{\omega_s}{2H}\frac{EV}{X}\sin\Theta. \tag{8b}$$

The eddy-diffusivity tensor $\mathcal{D}$ is defined as:

$$\mathcal{D}(\Theta, \Omega, t) = \frac{\omega_s^2}{4H^2}\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle P_{\mathrm{m}}(t)\,P_{\mathrm{m}}(s)\rangle \mathcal{G}_{\mathrm{d}}\;\mathrm{d}s\,\mathrm{d}\Theta'\,\mathrm{d}\Omega'. \tag{9}$$

The deterministic Green's function $\mathcal{G}_{\mathrm{d}}\,(\mathbf{x}, \mathbf{x}'; t - s)$ satisfies:

$$\frac{\partial \mathcal{G}_{\mathrm{d}}}{\partial s} + \langle \mathbf{v}\rangle \cdot \nabla_{\mathbf{x}'}\mathcal{G}_{\mathrm{d}} = -\delta(\mathbf{x}' - \mathbf{x})\delta(s - t), \tag{10}$$

where

$$\mathbf{x} = (\Theta, \Omega), \quad \langle \mathbf{v}\rangle = \begin{bmatrix} \Omega - \omega_{\mathrm{s}} \\ \frac{\omega_s}{2H}\left(\langle P_{\mathrm{m}}\rangle - \frac{EV}{X}\sin\Theta\right)\end{bmatrix}. \tag{11}$$