Software Library for Scalable Multi-Physics Multi-Scale Network Simulation:

Application to Water Distribution Systems

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Abstract

DMNetwork is a software library for scalable network simulation included in the scientific computing library PETSc. We use **DMNetwork** for the development of a scalable simulator for water distribution systems that includes high-fidelity physical model couplings.

System Level Simulation

System Level Simulation (SLS): collection of techniques to simulate the bahaviour of large cyber-physical systems.

- SLS involves multi-physics models
- SLS is frequently cross-disciplinary
- SLS is generally built upon a hierarchy of models

We would like to build a set of abstractions on top of PETSc to ease simulation of multi-physics network systems.

DMNetwork

DMNetwork is a DM class built on top of DMPlex for easily expressing and managing unstructured network problems.

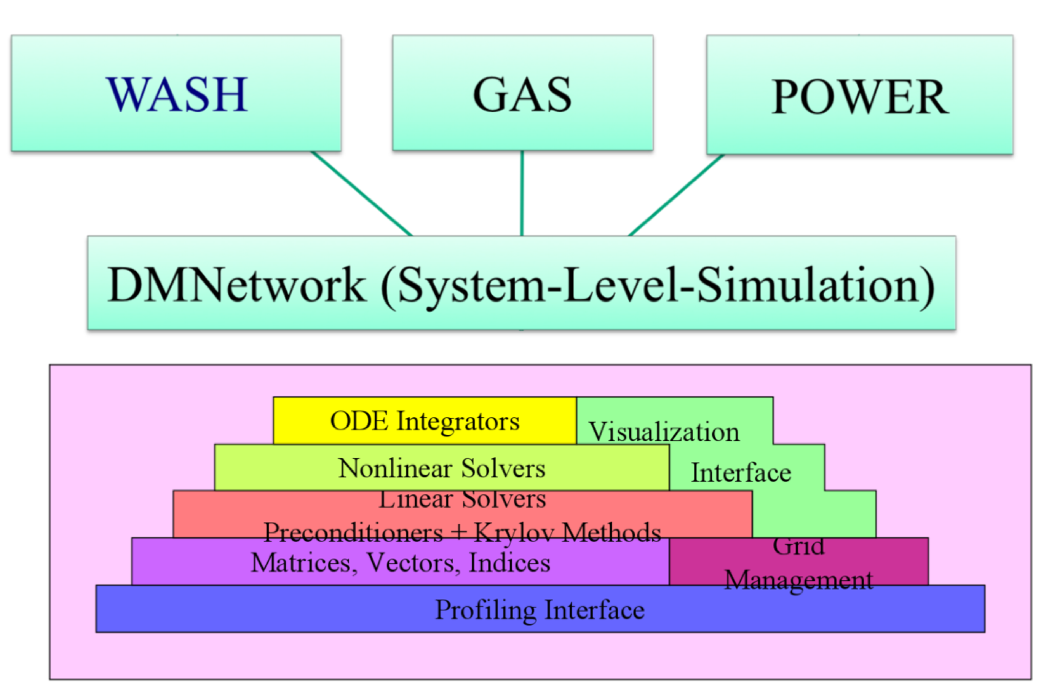

Figure 1: DMNetwork hierarchy.

Shock waves in water networks

The physical model of water flow in a pipe can be described with the following set of P.D.E's:

$$\frac{\partial Q}{\partial t} + gA\frac{\partial H}{\partial x} + RQ\,|Q| = 0 \qquad (1)$$

$$a^2\frac{\partial Q}{\partial x} + gA\frac{\partial H}{\partial t} = 0 \qquad (2)$$

which are the momentum and continuity equations for the flow $Q(x,t)$ and pressure $H(x,t)$. Pipes can be connected in a networked fashion where their boundaries have to satisfy:

$$\sum Q_i = 0 \quad \forall i \qquad (3)$$

$$H_i = H_j \quad \forall i \neq j \qquad (4)$$

Our simulator performs a transient analysis to determine maximum pressures and flows along a network after some disturbance has occurred.

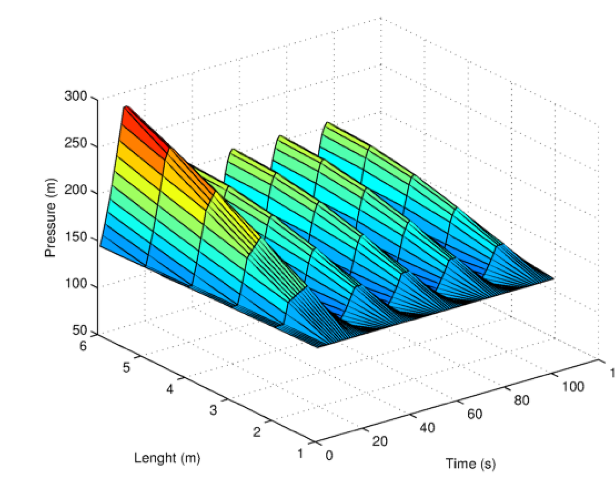

Figure 2: Wave propagation in pipe.

A sudden surge in pressure can lead to the burst of a pipe or the mis-functioning of a pump.

Structure of network

In the water network we define two main components:

- **Junction**: defines the boundary conditions of the pipe as well as the structure of the network.
- **Pipe**: where the physical process takes place.

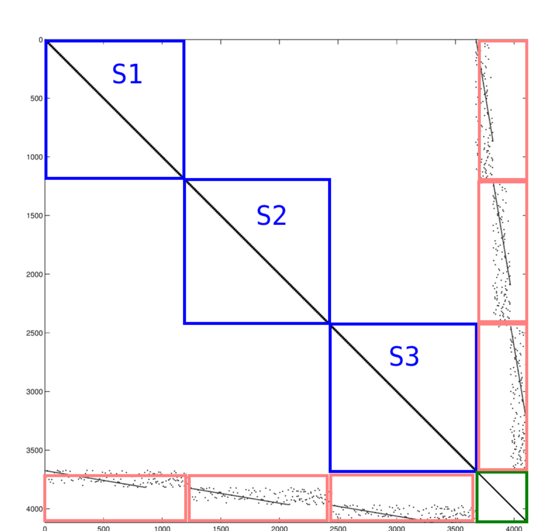

Figure 3: Structure of the Jacobian

We have provided abstractions to load independent networks and glue them together.

In Fig. 3 we can observe how the problem is structured. The submatrices S_i (in blue) contain the interior pipe equations.

The submatrix J (in green) is a matrix that contains all the junction or boundary equations.

We found that submatrix J is very ill conditioned while S_i are very easy to solve. Using PETSc FieldSplit with a Multiplicative Schwarz method we divided the problem in two fields.

- **Field 1:** including the submatrix J.
- **Field 2:** including all the problem domain.

For Field 1 we used LU decomposition whereas for Field 2 a block Jacobi preconditioner was employed. One has to take in account that after the discretization of the PDE's, the submatrices S_i become very large whereas J is relatively small.

Results

We have generated realistic water networks and tested the scalability of our simulator in ANL's CETUS.

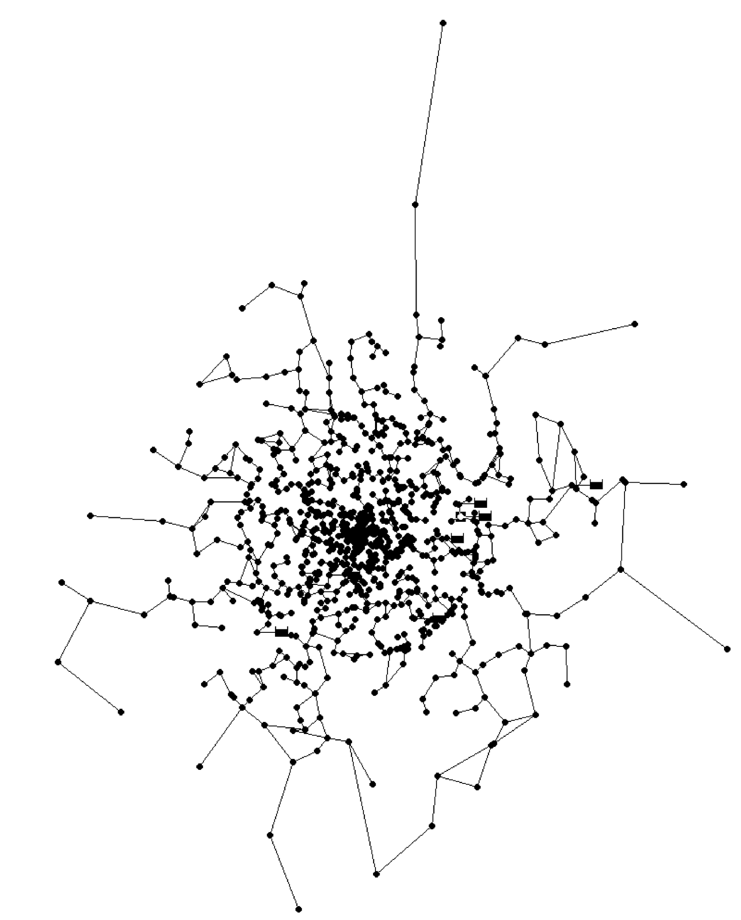

Figure 4: Artificially generated network

We have performed a series of experiments where several of these networks where joined and the scalability of the simulator was tested. In the following table we show the time spent in the time stepping for an increasing problem size.

cores	variables	time (sec)
64	4 M	8.85E1
128	8 M	1.08E2
256	16 M	1.13E2
512	32 M	1.18E2
1024	64 M	1.48E2
2048	126 M	1.67E2
4096	253 M	2.28E2

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