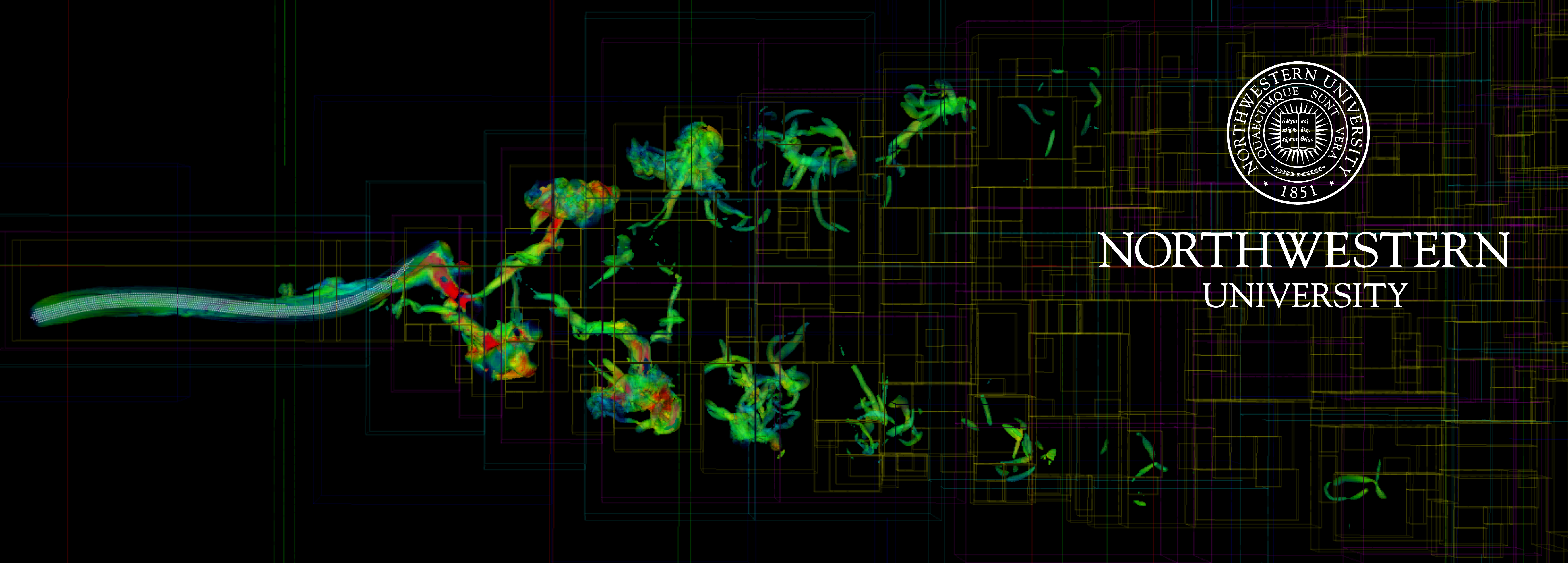


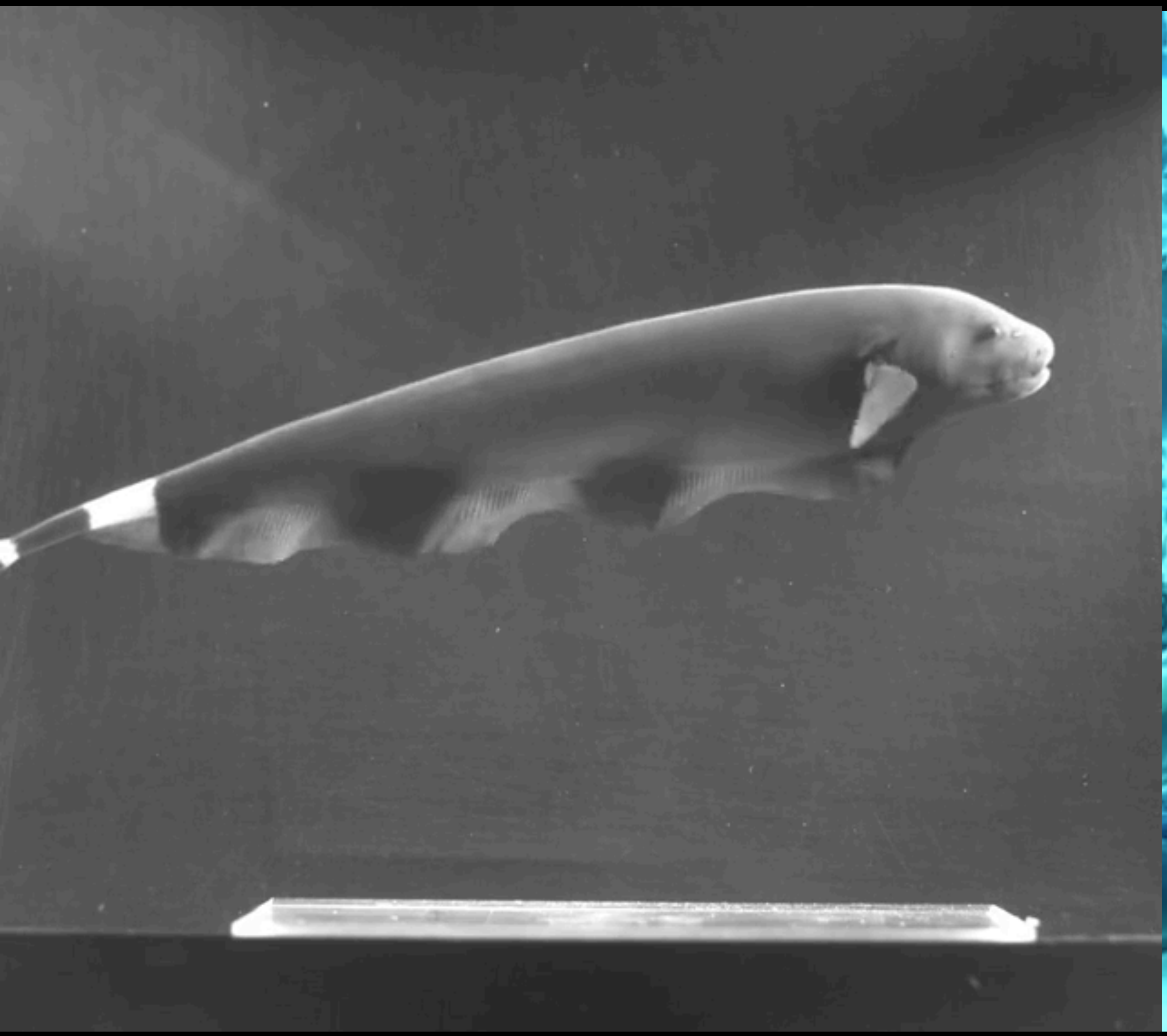
NORTHWESTERN UNIVERSITY

Fast Computation of Fully Resolved Neuromechanically Simulated Locomotion

Namu Patel & Neelesh A. Patankar

Neuromechanically driven locomotion

Neuromechanically driven locomotion



Malcolm MacIver at Northwestern University

Video from: https://www.youtube.com/watch?v=LDrvbr_CbhE&spfreload=10

Melina Hale at University of Chicago

Neural activation wave → Muscle contraction wave → Kinematic deformation wave

Equations for Fluid-Structure Interaction

Immersed Boundary Equations for Fluid-Structure Interaction

momentum
$$\rho \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t}(\mathbf{x},t) = -\nabla p(\mathbf{x},t) + \mu \nabla^2 \mathbf{u}(\mathbf{x},t) + \mathbf{f}(\mathbf{x},t)$$

continuity
$$\nabla \cdot \mathbf{u}(\mathbf{x},t) = 0$$

Immersed Boundary Equations for Fluid-Structure Interaction

momentum $$\rho \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t}(\mathbf{x},t) = -\nabla p(\mathbf{x},t) + \mu \nabla^2 \mathbf{u}(\mathbf{x},t) + \mathbf{f}(\mathbf{x},t)$$

continuity $$\nabla \cdot \mathbf{u}(\mathbf{x},t) = 0$$

Lagrangian point velocity $$\frac{\partial \mathbf{X}}{\partial t}(\mathbf{s},t) = \int_{\mathcal{U}_b} \mathbf{u}(\mathbf{x},t)\delta(\mathbf{x} - \mathbf{X}(\mathbf{s},t))\mathrm{d}\mathbf{x}$$

Immersed Boundary Equations for Fluid-Structure Interaction

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force spreading $$\mathbf{f}(\mathbf{x},t) = \int_{\mathcal{U}_b} \mathbf{F}(\mathbf{s},t)\delta(\mathbf{x} - \mathbf{X}(\mathbf{s},t))\,\mathrm{d}\mathbf{x}$$

Forcing Term

$$\mathbf{f}(\mathbf{x},t) = \int_{\mathcal{U}_b} \mathbf{F}(\mathbf{s},t)\delta(\mathbf{x}-\mathbf{X}(\mathbf{s},t))\,\mathrm{d}\mathbf{x}$$

$$\mathbf{F}(\mathbf{s},t) = \mathbf{F}_E(\mathbf{s},t) + \mathbf{F}_C(\mathbf{s},t)$$

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Elastic body

$$\mathbf{F}_E^n = \underbrace{K_s\left[\mathbf{X}^n - \mathbf{X}_0^n\right]}_{\text{Spring force}} + \underbrace{K_b\left[\kappa(\mathbf{X}^n) - \kappa_p^n\right]}_{\text{Beam force}}$$

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Stiff body

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Stiff body → $K_s > O(\Delta s)^{-1}$, $K_b > O(\Delta s)^{-5}$

Forcing Term

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$$\rho \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t}(\mathbf{x},t) = -\nabla p(\mathbf{x},t) + \mu \nabla^2 \mathbf{u}(\mathbf{x},t) + \mathbf{f}(\mathbf{x},t)$$

$$\mathbf{f}(\mathbf{x},t) = \int_{\mathcal{U}_b} \mathbf{F}_C(\mathbf{s},t)\delta(\mathbf{x} - \mathbf{X}(\mathbf{s},t))\mathrm{d}\mathbf{x}$$

Solid, deforming body

$$\mathbf{f}(\mathbf{x},t) = \lambda(\mathbf{x},t) \quad \text{Lagrange multiplier}$$

$$\mathbf{U}_d = \mathbf{U}_r + (\mathbf{W}_r \times \mathbf{R}) + \mathbf{U}_k \quad \in \Omega_r$$

A.P.S. Bhalla, B.E. Griffith, N.A. Patankar (2013) **PLoS Comp Biol**
A.P.S. Bhalla, R. Bale, B.E. Griffith, N.A. Patankar (2013) **JCP**

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Directly impose kinematic deformations, NOT the deformation velocity

A.P.S. Bhalla, B.E. Griffith, N.A. Patankar (2013) **PLoS Comp Biol**
A.P.S. Bhalla, R. Bale, B.E. Griffith, N.A. Patankar (2013) **JCP**

Constraint Algorithm

Constraint Algorithm

1. Predict the position of the Lagrangian structure

Constraint Algorithm

1. Predict the position of the Lagrangian structure

2. Solve the fluid equations without kinematic constraints

Constraint Algorithm

1. Predict the position of the Lagrangian structure
2. Solve the fluid equations without kinematic constraints
3. Impose the deformation kinematics

Constraint Algorithm

1. Predict the position of the Lagrangian structure
2. Solve the fluid equations without kinematic constraints
3. Impose the deformation kinematics
4. Correct the fluid velocity and pressure and position of Lagrangian structure

Constraint Algorithm

Step 1. Predict the position of the constrained body

$$\mathbf{X}_p^{n+1} = \mathbf{X}^n + \Delta t \mathbf{U}_d^n$$

Constraint Algorithm

Step 2. Solve the fluid equations without kinematic constraints

$$\rho\left(\frac{\mathbf{u}_p^{n+1}-\mathbf{u}^n}{\Delta t}+[\mathbf{u}\cdot\nabla\mathbf{u}]_p^{n+1/2}\right)=-\nabla p_p^{n+1/2}+\mu\nabla^2\mathbf{u}_p^{n+1}$$

$$\nabla\cdot\mathbf{u}_p^{n+1}=0$$

Constraint Algorithm

Step 3. Interpolate the fluid velocity using discrete Delta functions

$$\mathbf{U}_p^{n+1} = R[\mathbf{X}_p^{n+1}]\mathbf{u}_p^{n+1}$$

Constraint Algorithm

Step 3a. Interpolate the fluid velocity using discrete Delta functions

$$\mathbf{U}_p^{n+1} = R[\mathbf{X}_p^{n+1}]\mathbf{u}_p^{n+1}$$

$$R[\mathbf{X}_p^{n+1}]\mathbf{u}_p^{n+1} \approx \int_{\mathcal{U}_b} \mathbf{u}_p^{n+1}\delta(\mathbf{x}-\mathbf{X}_p^{n+1})\,\mathrm{d}\mathbf{x}$$

Constraint Algorithm

Step 3b. Compute the rigid translational and rotational velocities

$$M_C \mathbf{U}_r^{n+1} = \sum_{\Omega_r} \rho \mathbf{U}_p^{n+1} \Delta V$$

$$\mathbf{I}_C^{n+1} \mathbf{W}_r^{n+1} = \sum_{\Omega_r} \rho \mathbf{R}^{n+1} \times \mathbf{U}_p^{n+1} \Delta V$$

$$\mathbf{R}^{n+1} = \mathbf{X}_p^{n+1} - \mathbf{X}_{p,com}^{n+1}$$

Constraint Algorithm

Step 3c. Given the prescribed kinematic shape $\chi(\mathbf{s},t)$, calculate the deformation shape $\mathbf{X}_k$, and velocity $\mathbf{U}_k$

Constraint Algorithm

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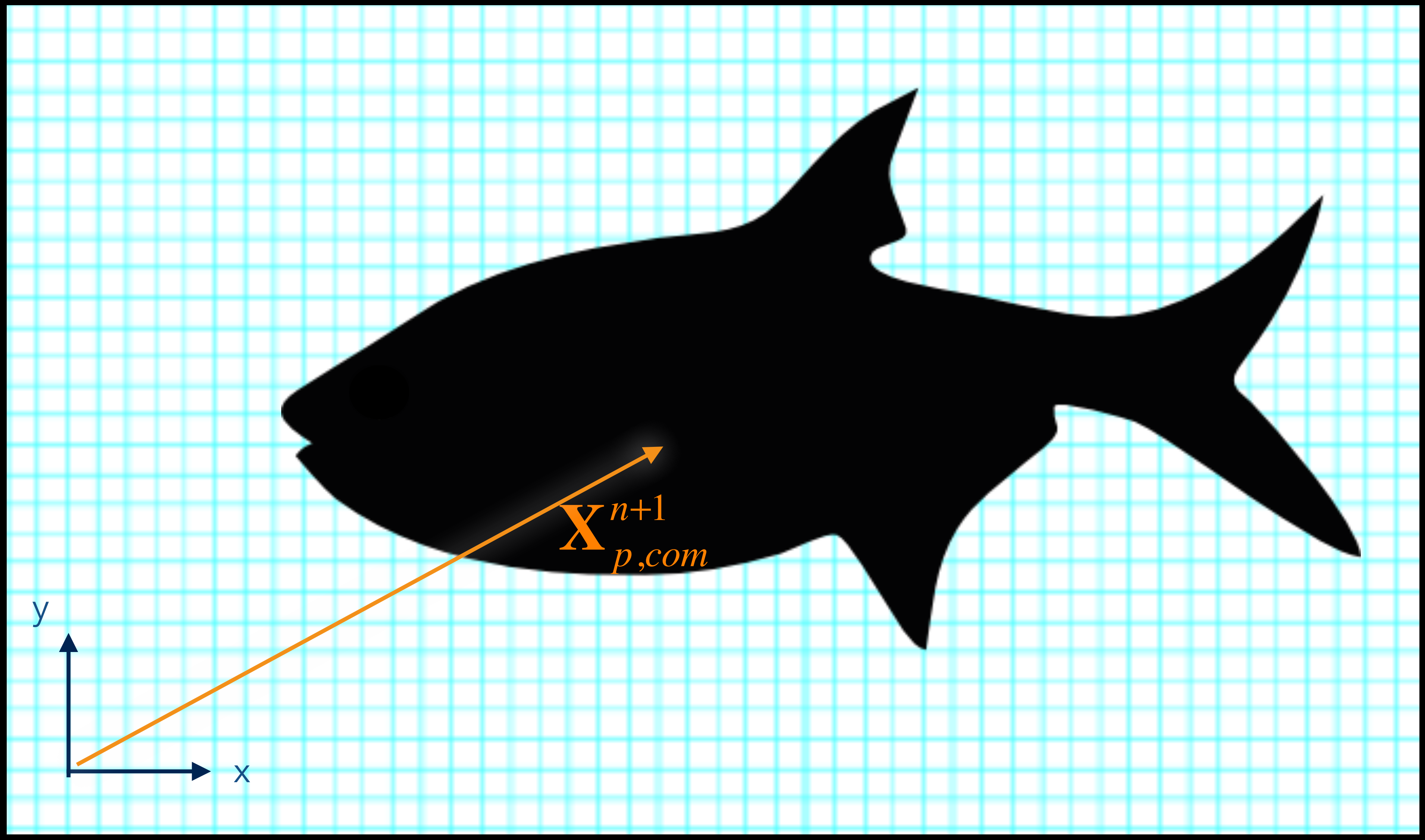

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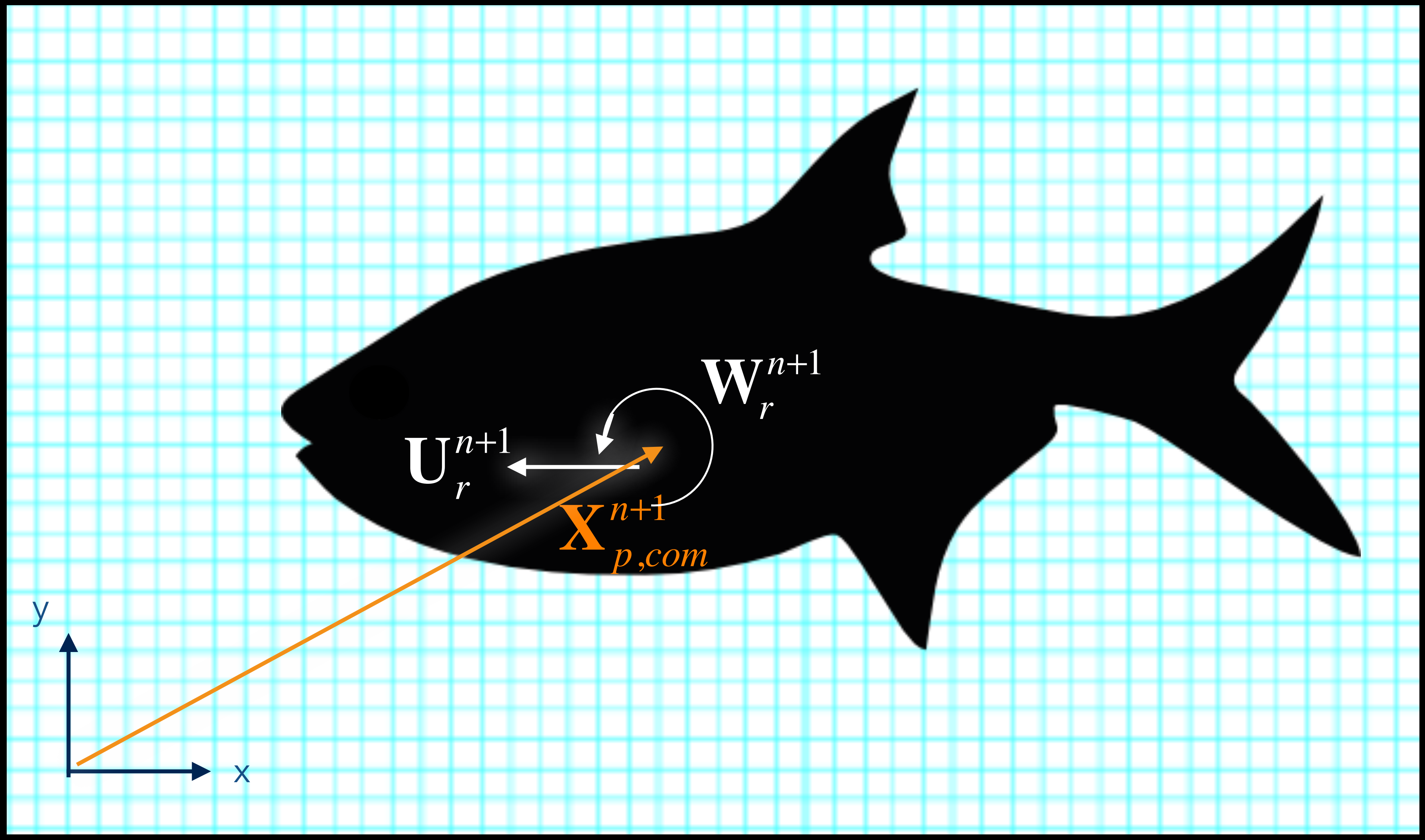

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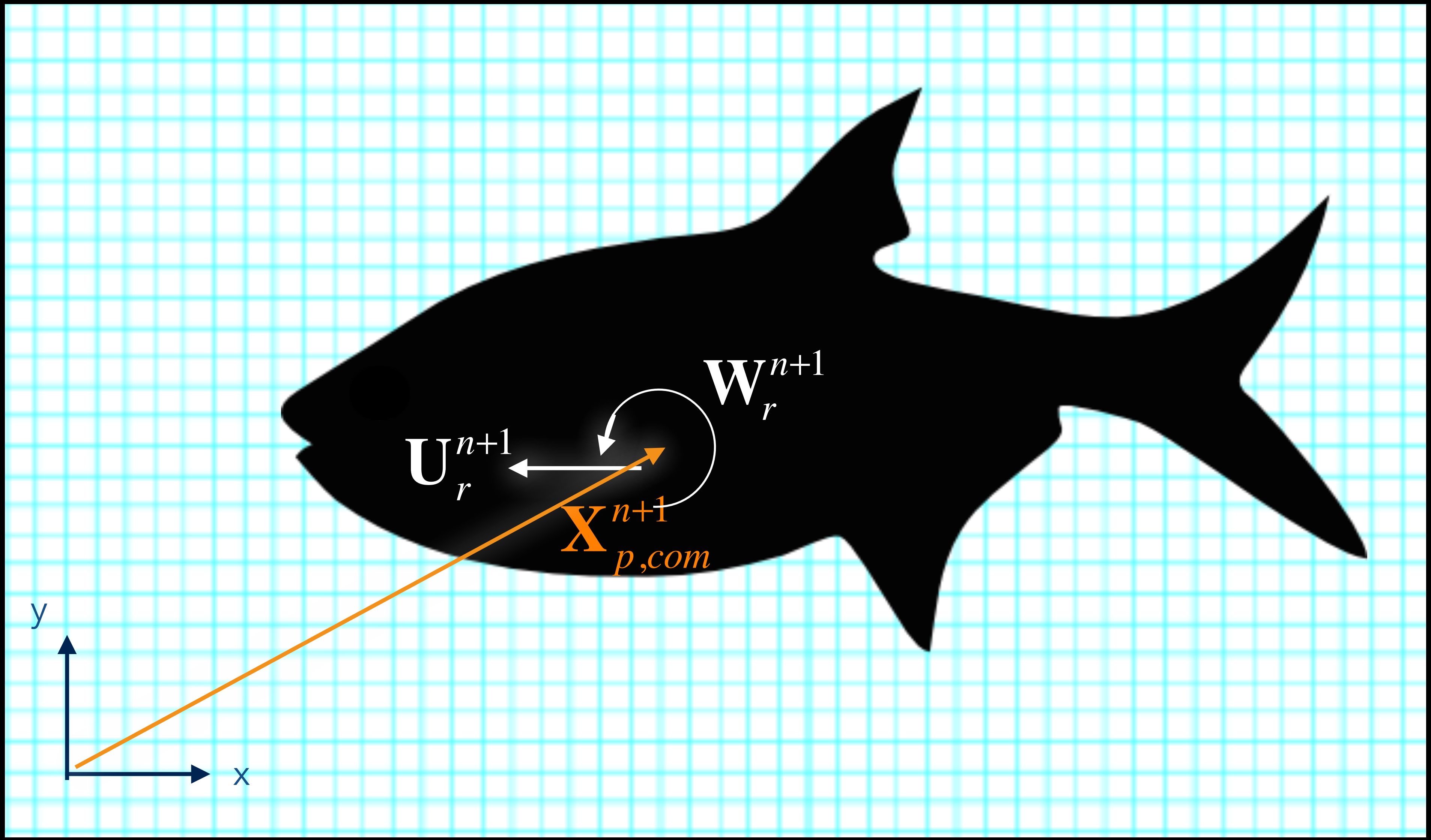


$$\mathbf{U}_d^{n+1} = \mathbf{U}_r^{n+1} + \mathbf{W}_r^{n+1} \times \mathbf{R}^{n+1} + \mathbf{U}_k^{n+1}$$

Constraint Algorithm

Step 4a. Compute the constraint force

$$\Delta \mathbf{U}_c^{n+1} = \mathbf{U}_d^{n+1} - \mathbf{U}_p^{n+1}$$

$$\mathbf{F}_C^{n+1} = \frac{\rho}{\Delta t} \Delta \mathbf{U}_c^{n+1}$$

Constraint Algorithm

Step 4b. Spread the corrected Lagrangian velocity

$$\rho \frac{\mathbf{u}^{n+1} - \mathbf{u}_p^{n+1}}{\Delta t} = -\nabla(p^{n+1/2} - p_p^{n+1/2}) + S[\mathbf{X}_p^{n+1}]\mathbf{F}_C^{n+1}$$

$$\nabla \cdot \mathbf{u}^{n+1} = 0$$

Constraint Algorithm

Step 4c. Correct the position of the Lagrangian structure

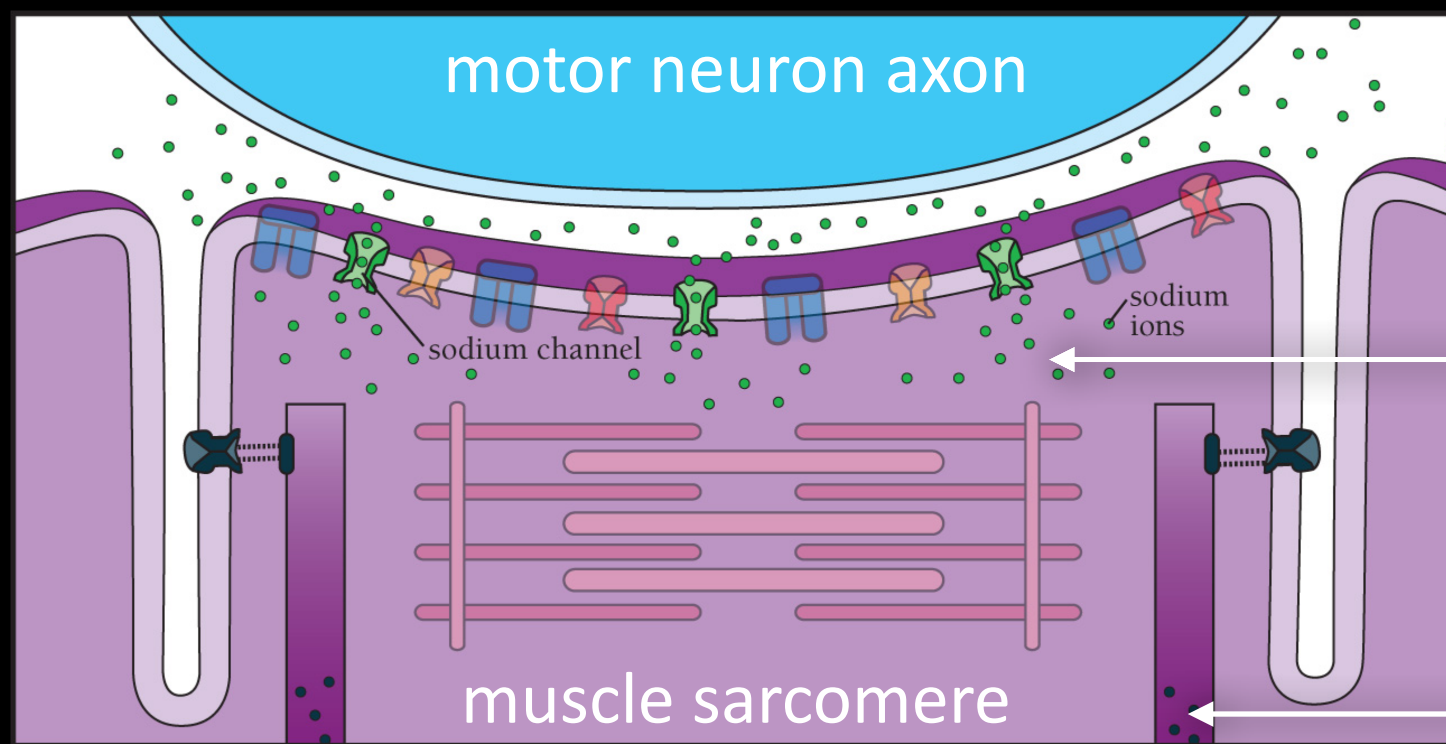
$$\mathbf{X}_{com}^{n+1} = \mathbf{X}_{com}^{n} + \frac{\Delta t}{2}\left(\mathbf{U}_r^n + \mathbf{U}_r^{n+1}\right)$$

$$\mathbf{X}^{n+1} = \mathbf{X}_{com}^{n+1} + \mathbf{X}_k^{n+1}$$

Neuromuscular Kinematics for Swimming

Specify kinematic constraints

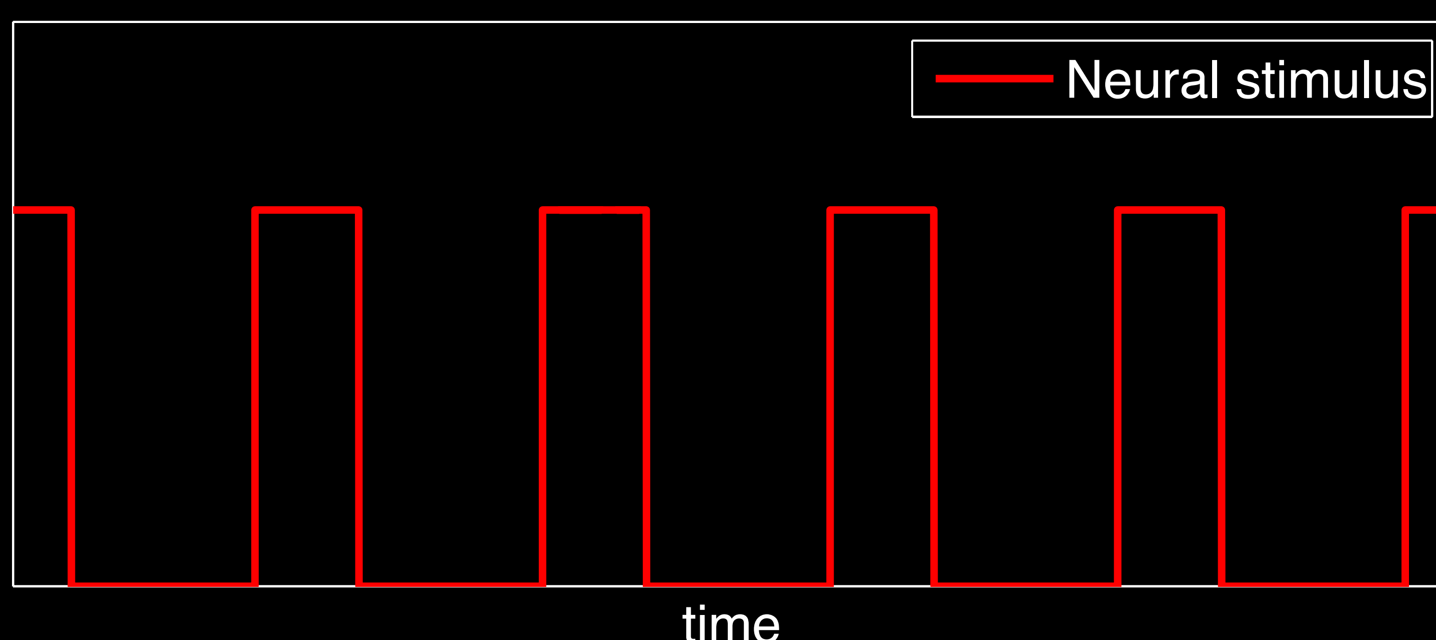
Neuromechanical model



cation influx
∴ depolarization

Ca^{2+}

External forcing

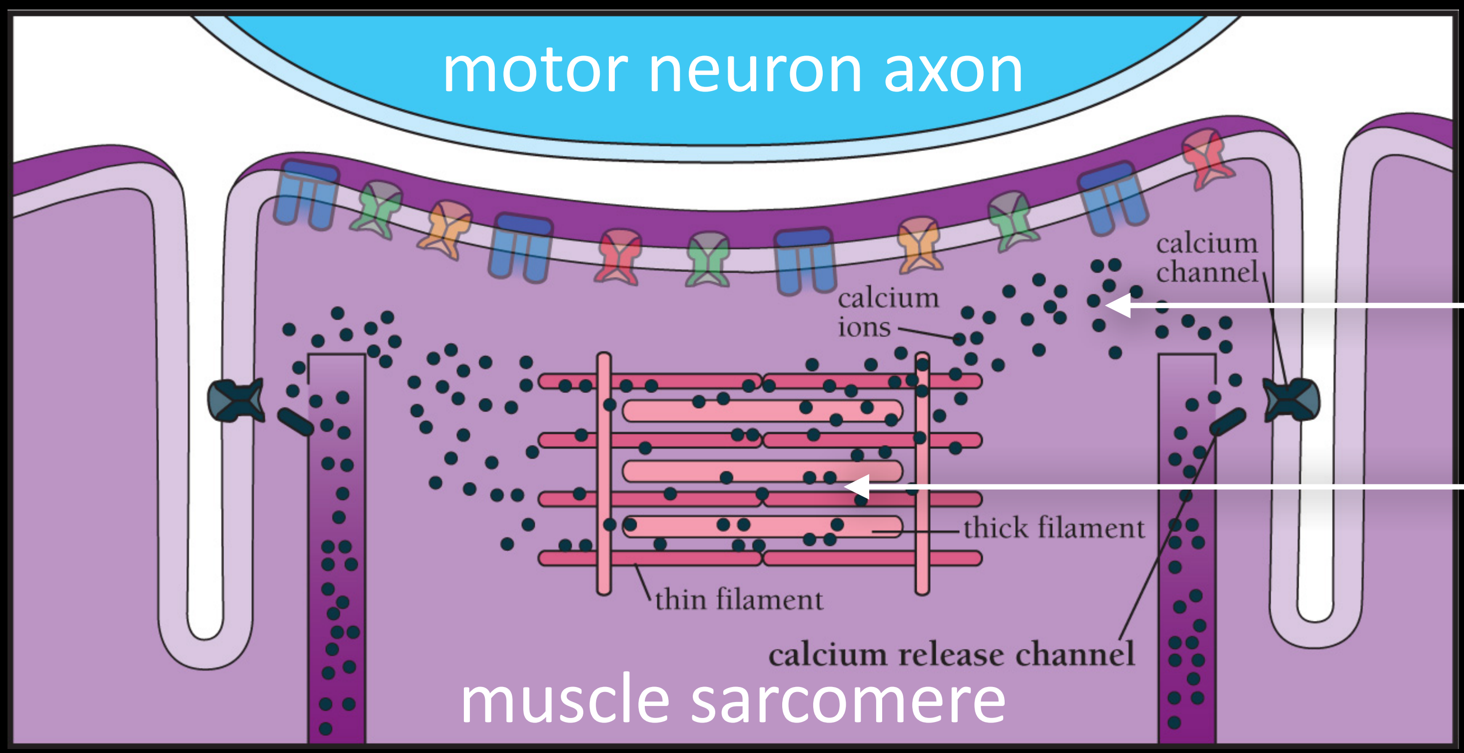


on

off

Specify kinematic constraints

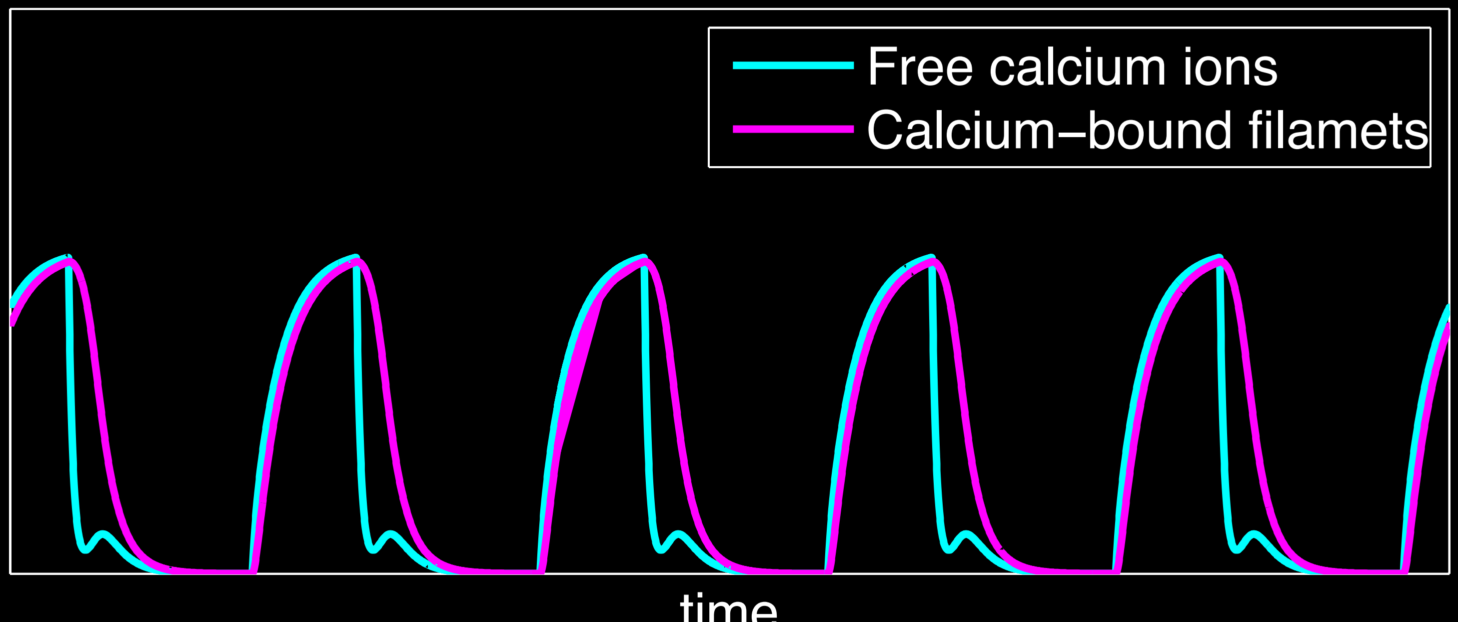
Neuromechanical model



Kinetic Equations

$$\frac{d\,Ca^{2+}_{free}}{dt} = f(Ca^{2+}_{free}, Ca^{2+}_{bound}, stim)$$

$$\frac{d\,Ca^{2+}_{bound}}{dt} = g(Ca^{2+}_{free}, Ca^{2+}_{bound})$$



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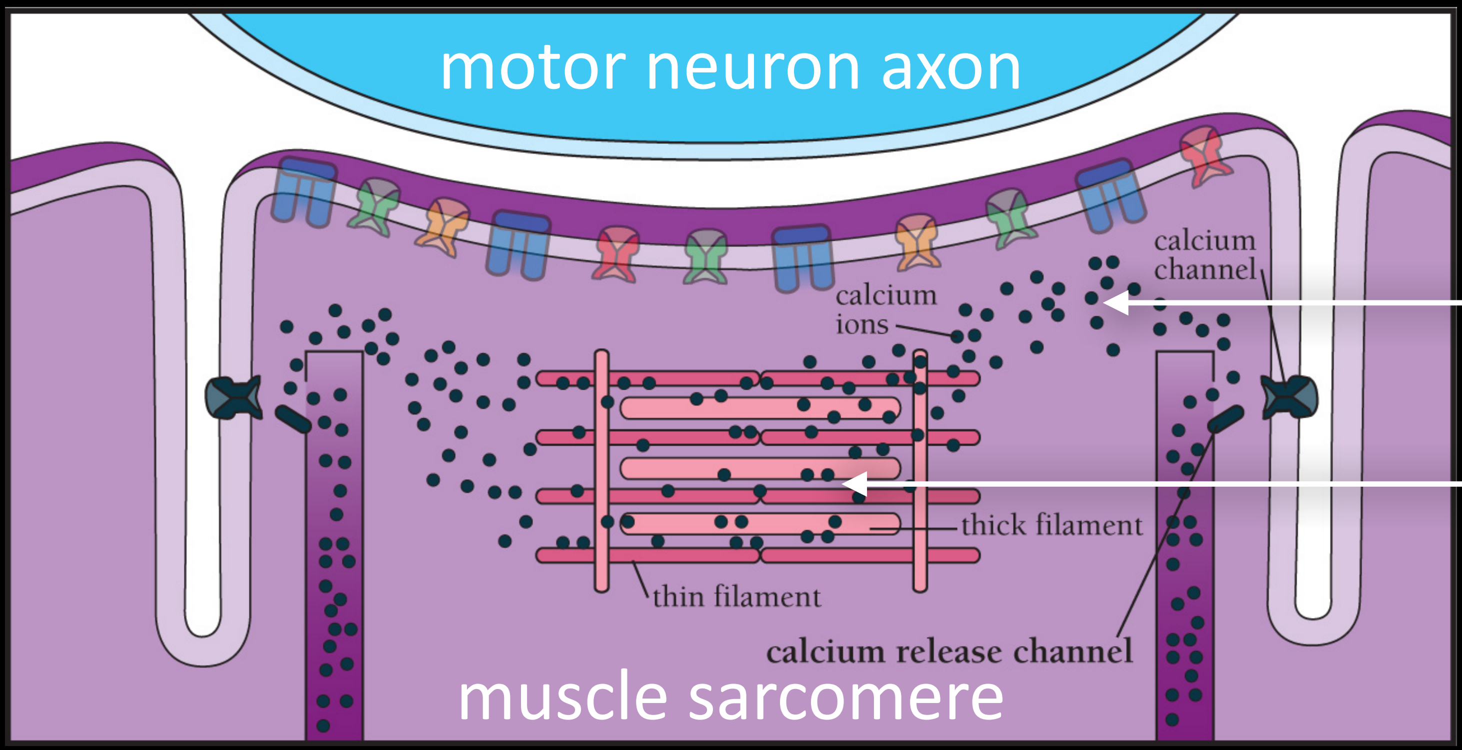
Specify kinematic constraints

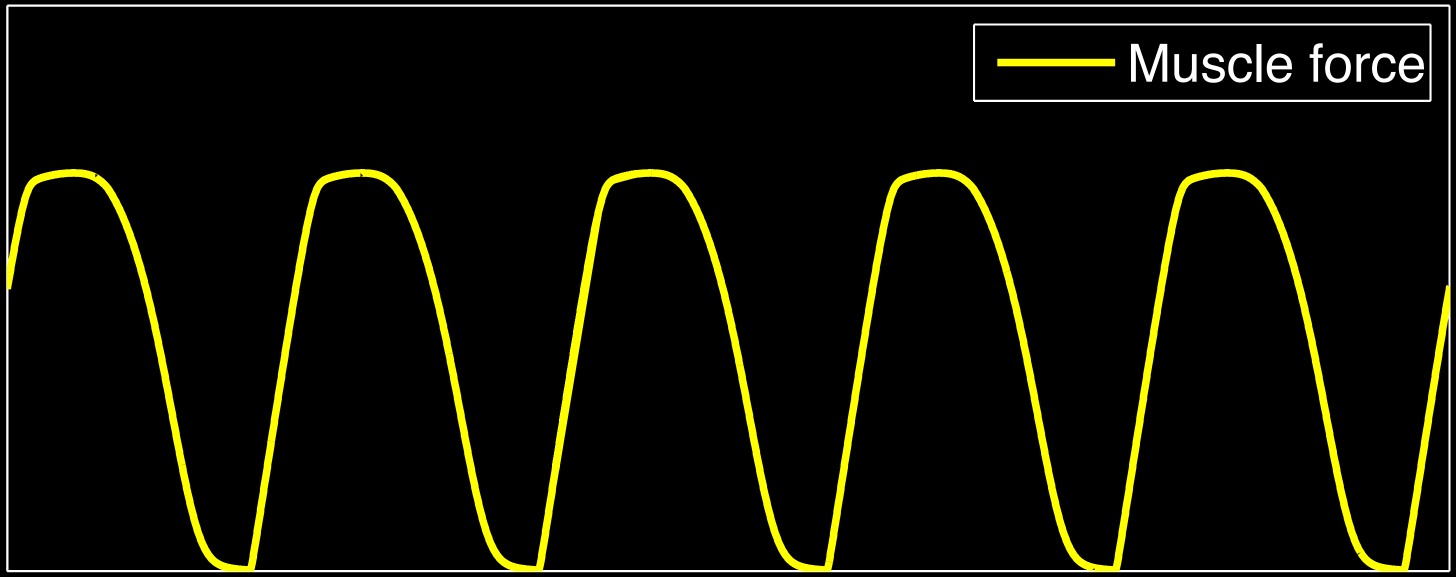
Neuromechanical model



Muscle Force

$$\frac{\mathrm{d}P}{\mathrm{d}t} = k(P_c - P)$$

$$P_c = P_c(l_{fiber}, v_{fiber}, Ca^{2+}_{bound})$$



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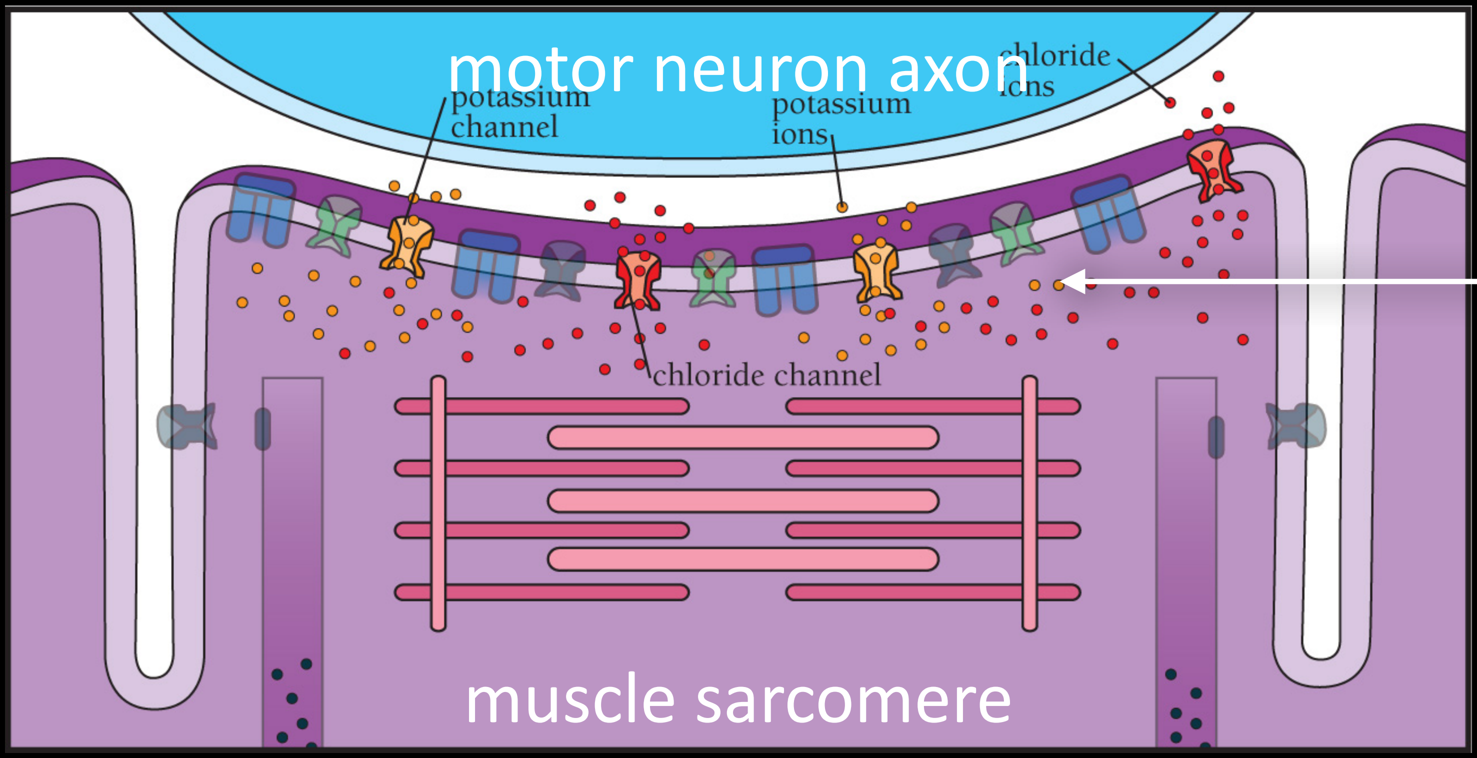
Specify kinematic constraints

Neuromechanical model

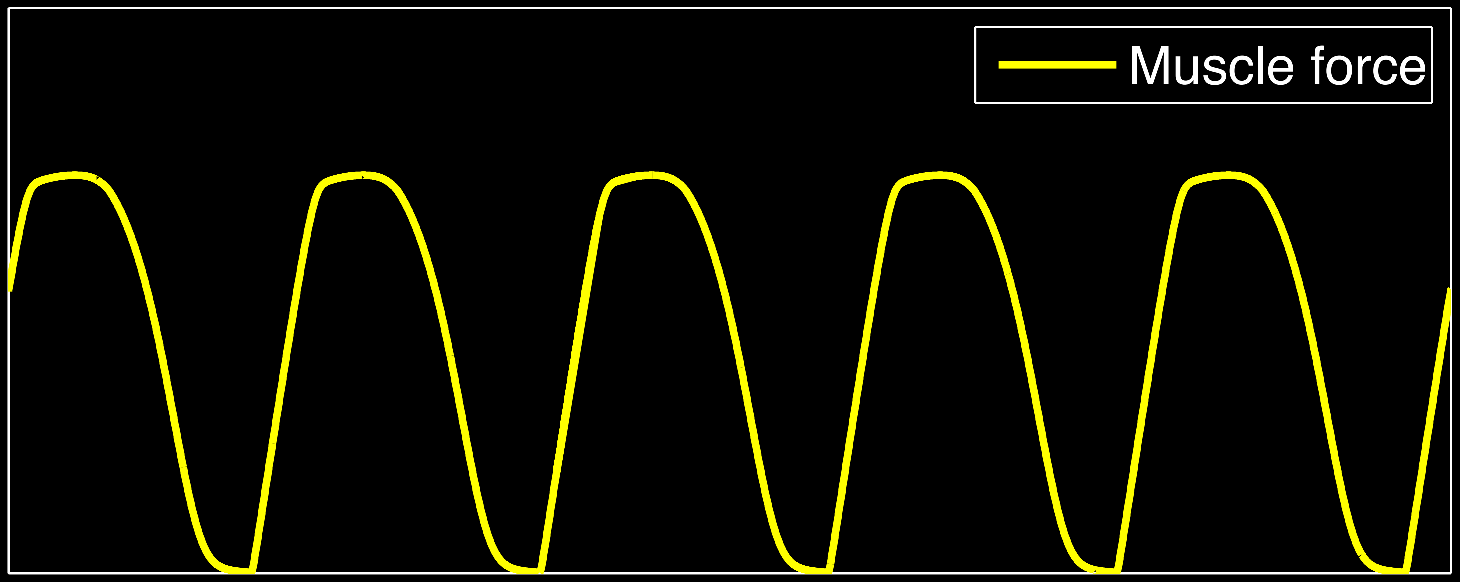


cation efflux
∴ repolarization

Muscle Force

$$\frac{\mathrm{d}P}{\mathrm{d}t} = k(P_c - P)$$

$$P_c = P_c(l_{fiber}, v_{fiber}, Ca^{2+}_{bound})$$



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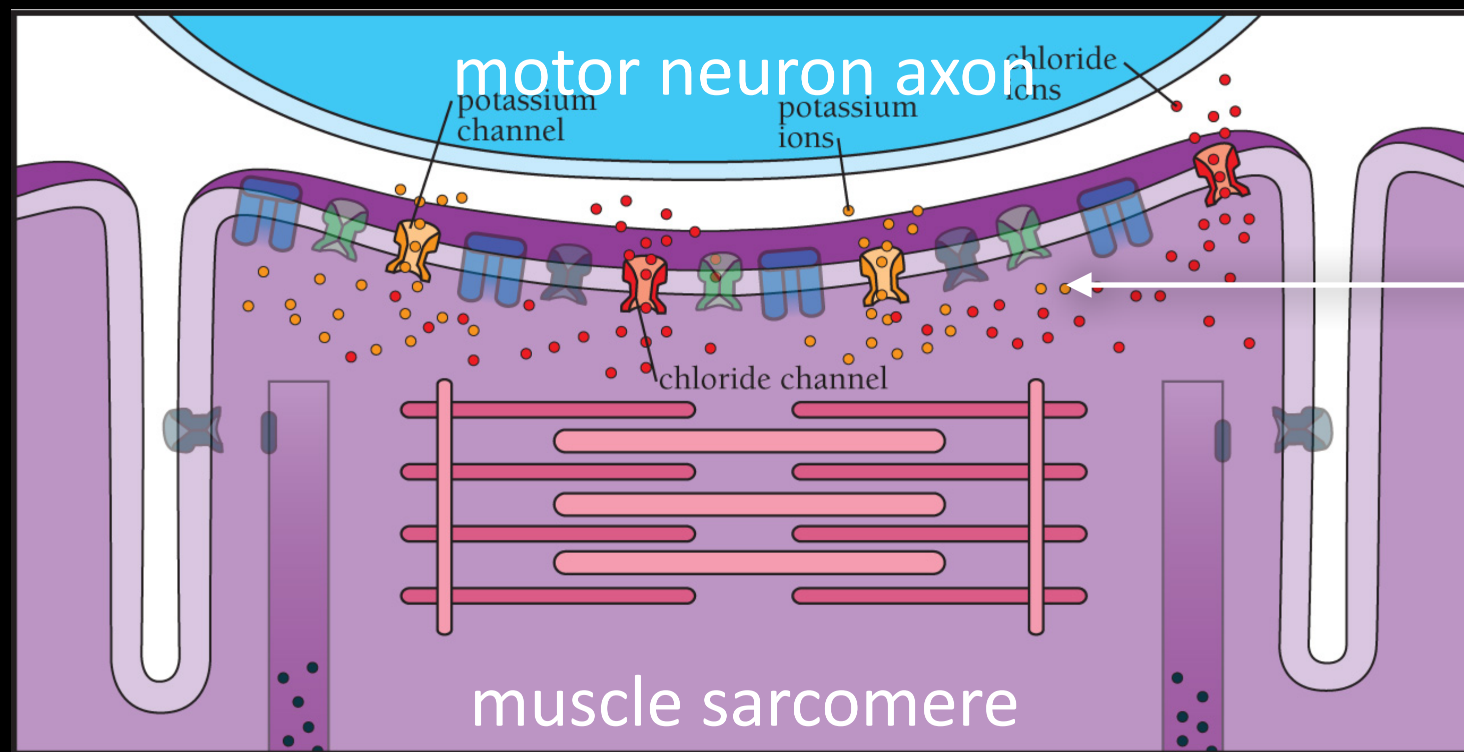
Specify kinematic constraints

Neuromechanical model

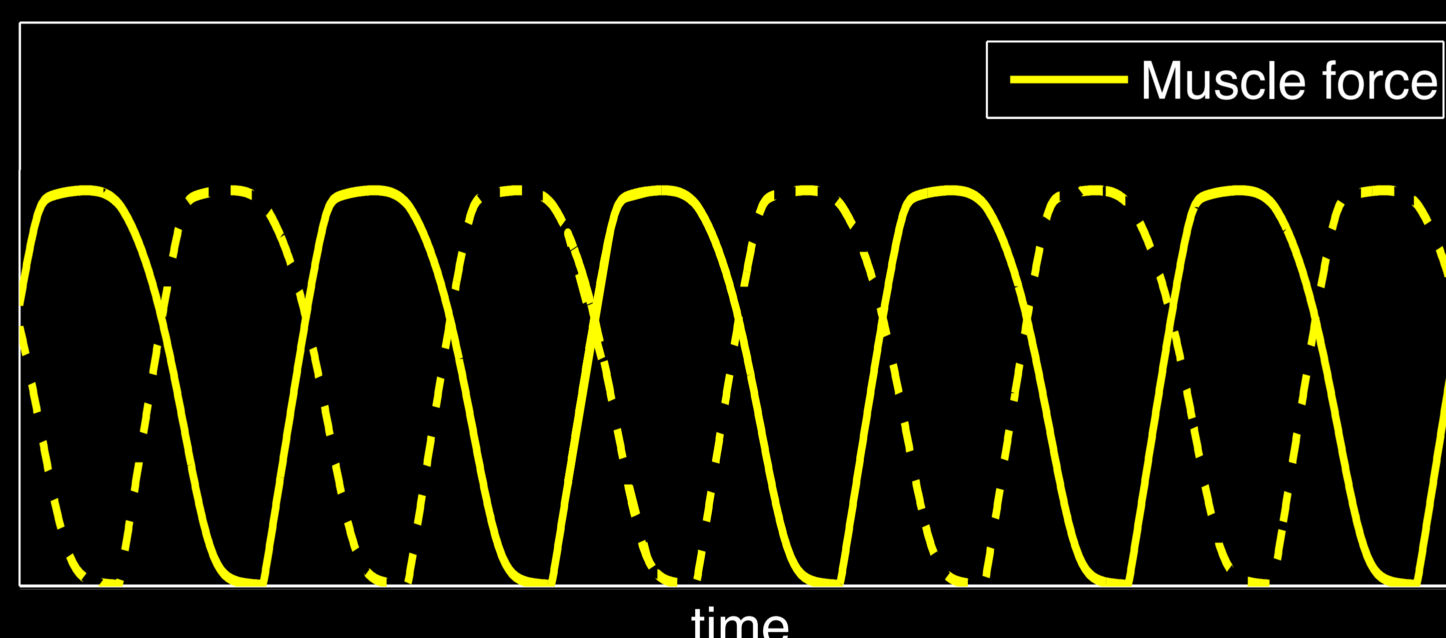


cation efflux
∴ depolarization

Muscle Force

$$\frac{\mathrm{d}P}{\mathrm{d}t} = k(P_c - P)$$

$$P_c = P_c(l, v, Ca^{2+}_{bound})$$



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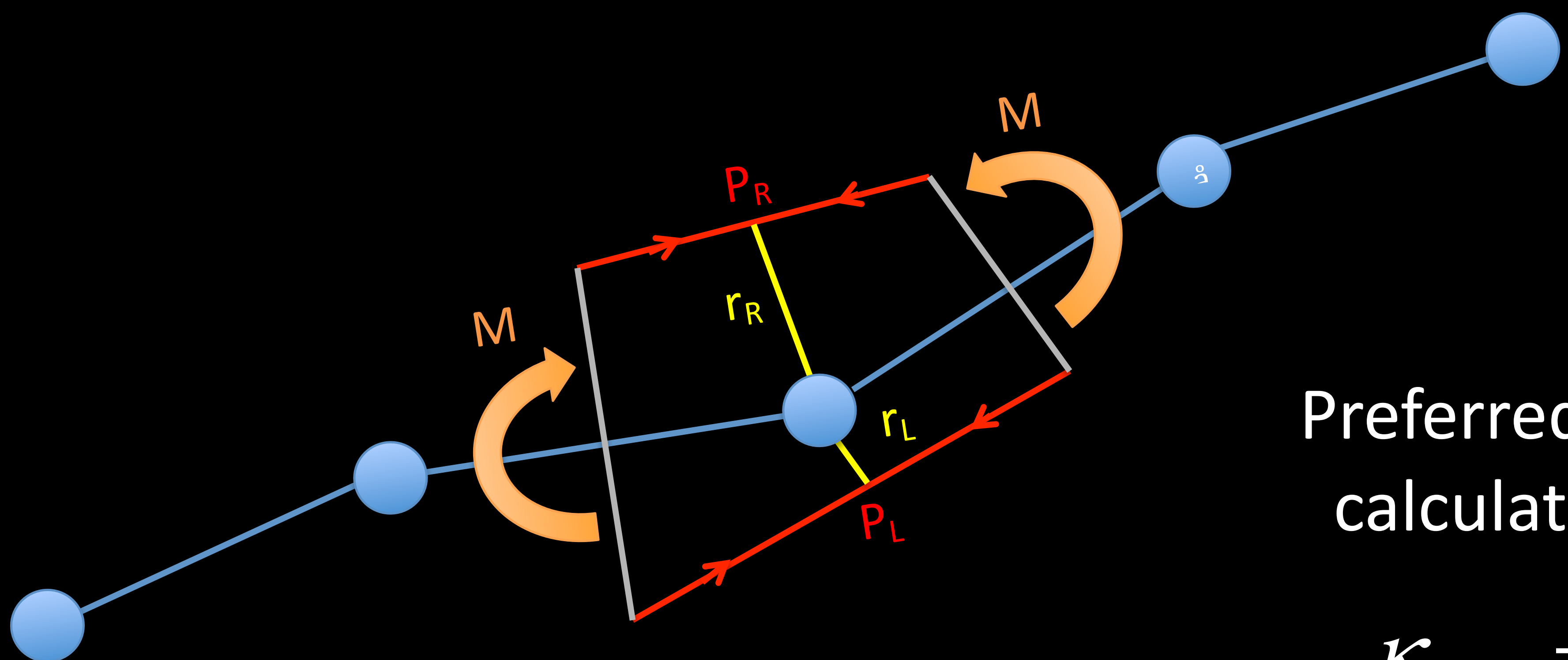
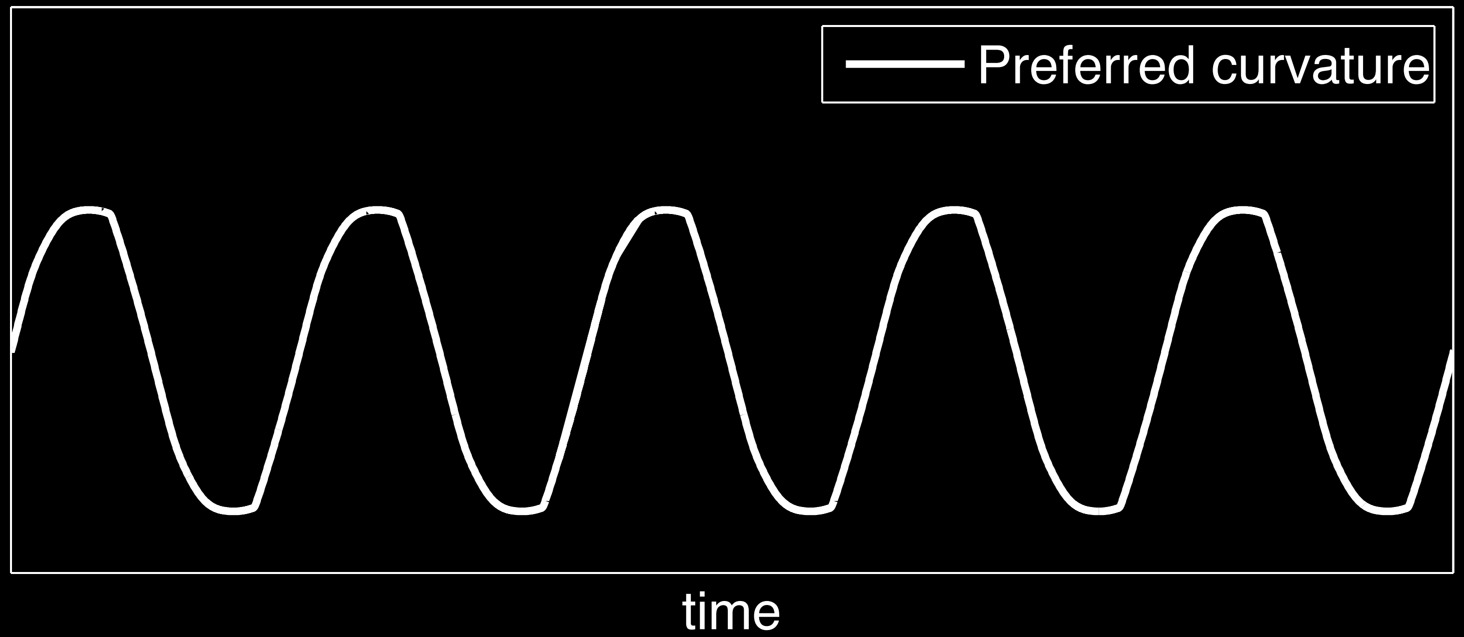
Specify kinematic constraints

Neuromechanical model

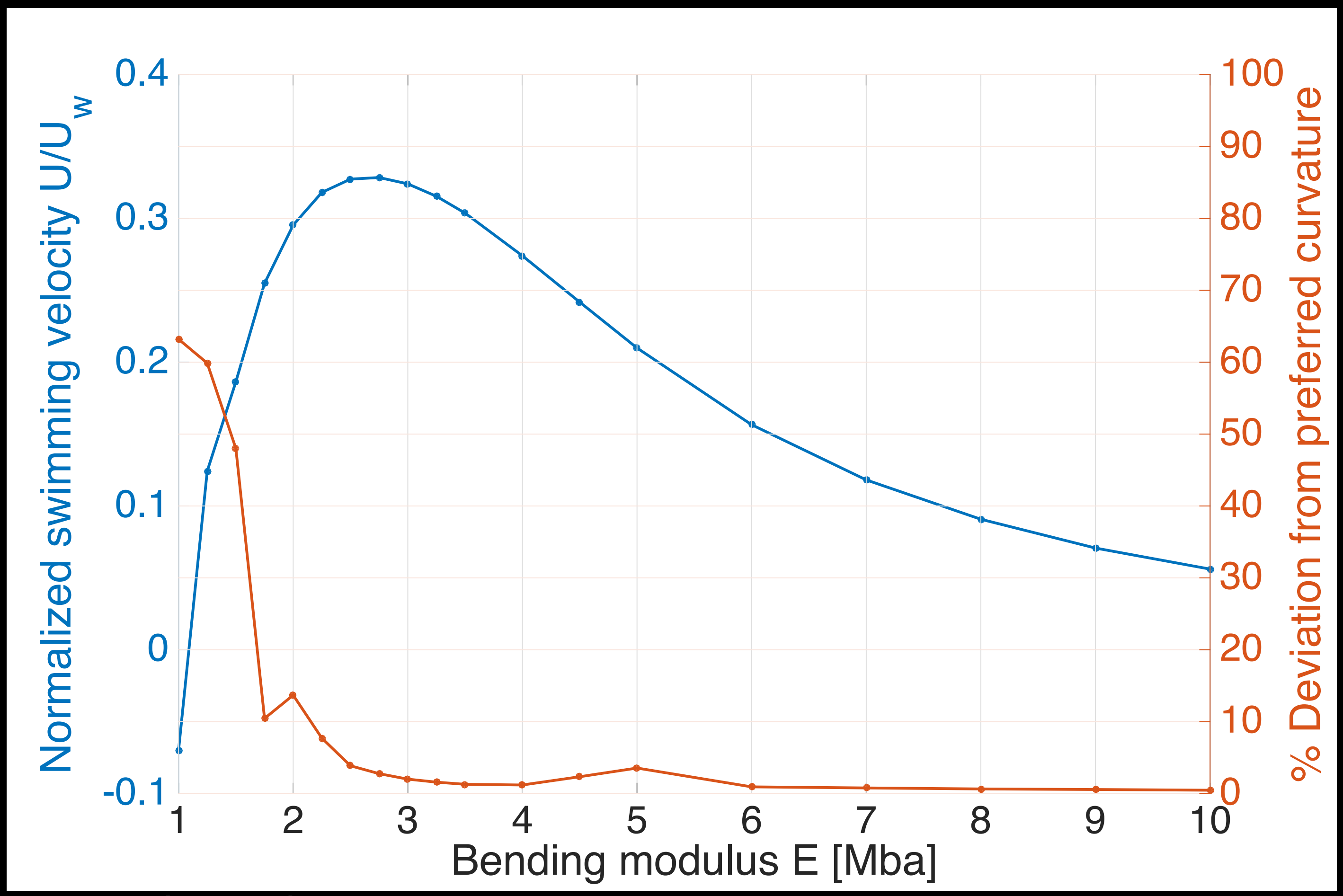


Preferred curvature used to calculate preferred shape

$$\kappa_{pref} = h(P_R, P_L, EI)$$



Reduced order fluid simulations for choosing EI



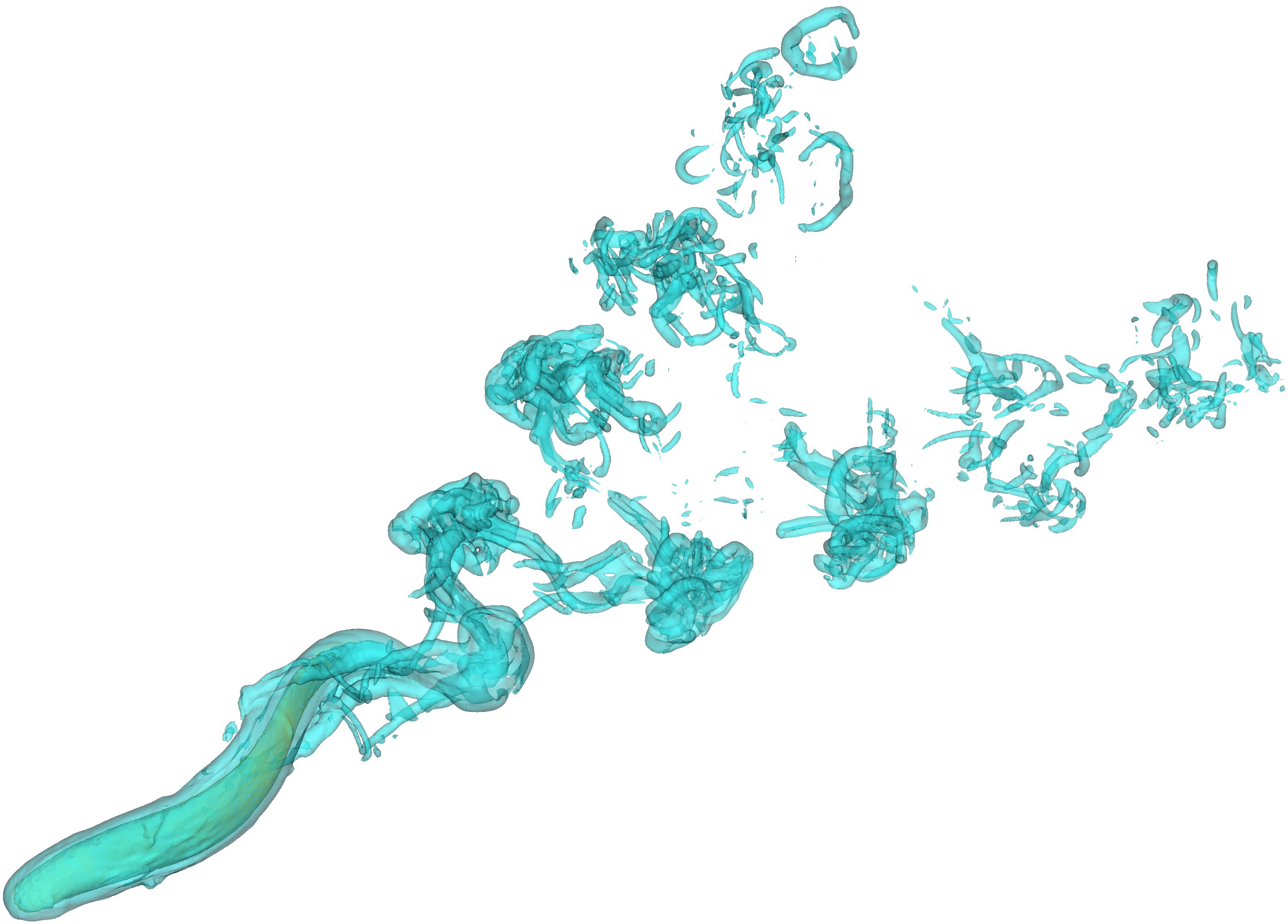
floppy swimmer

sufficiently stiff swimmer

$$\text{\% Deviation from preferred curvature} = 100 \max_{t_{ss}} \left(\frac{\left\| \frac{\partial \theta}{\partial s} - \kappa_p \right\|_2}{\left\| \frac{\partial \theta}{\partial s} \right\|_2} \right)$$

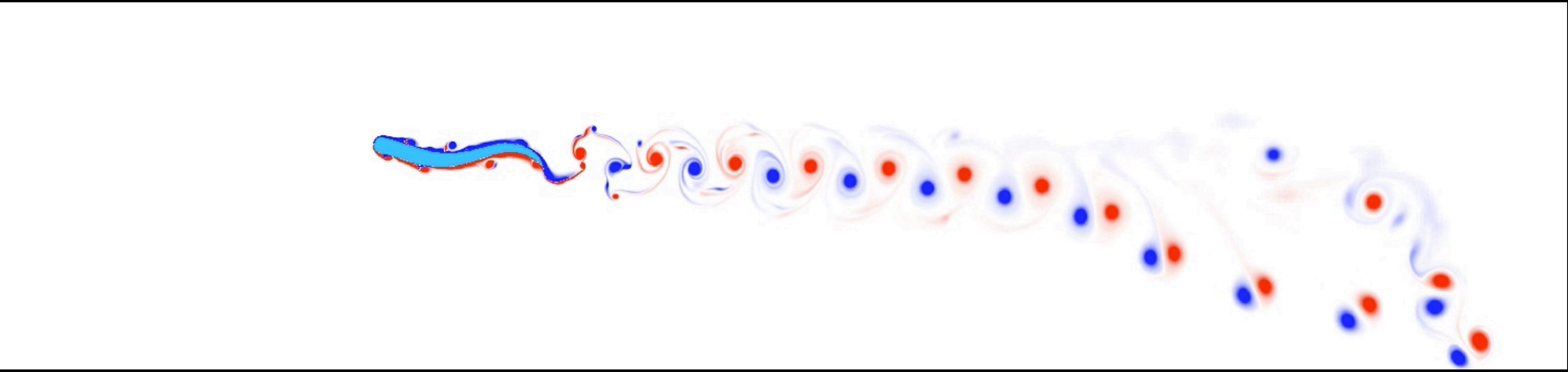
3D Neuromechanically Driven Locomotion

Fully resolved simulation of neuromechanically driven locomotion

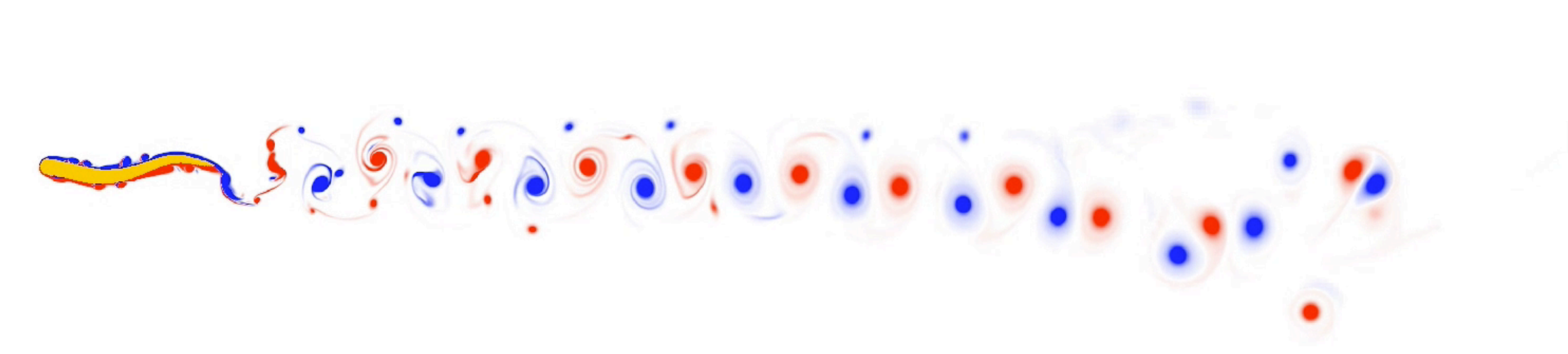


Muscle Co-contractions

Swimming with & without co-contractions

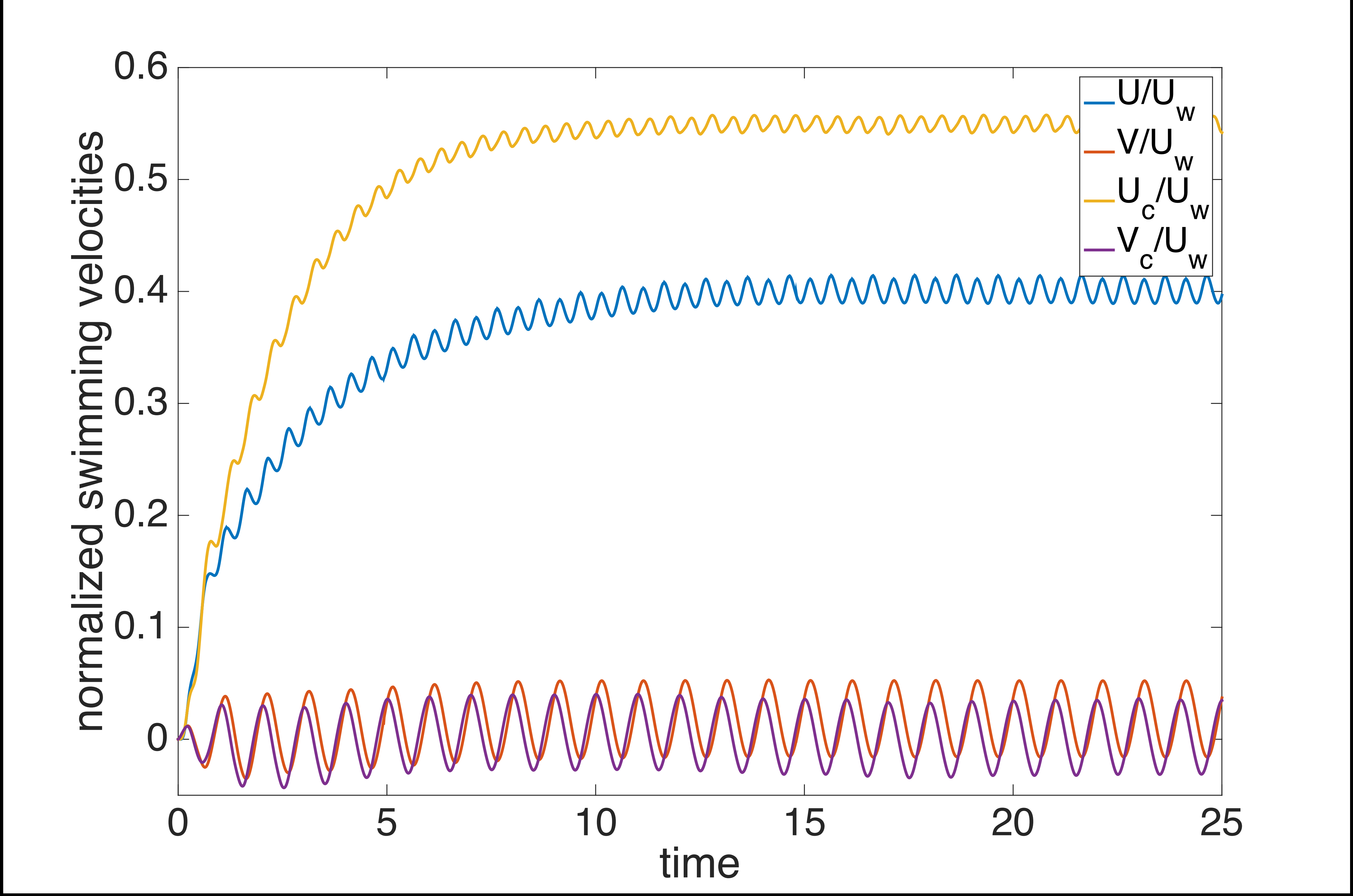


no co-contractions

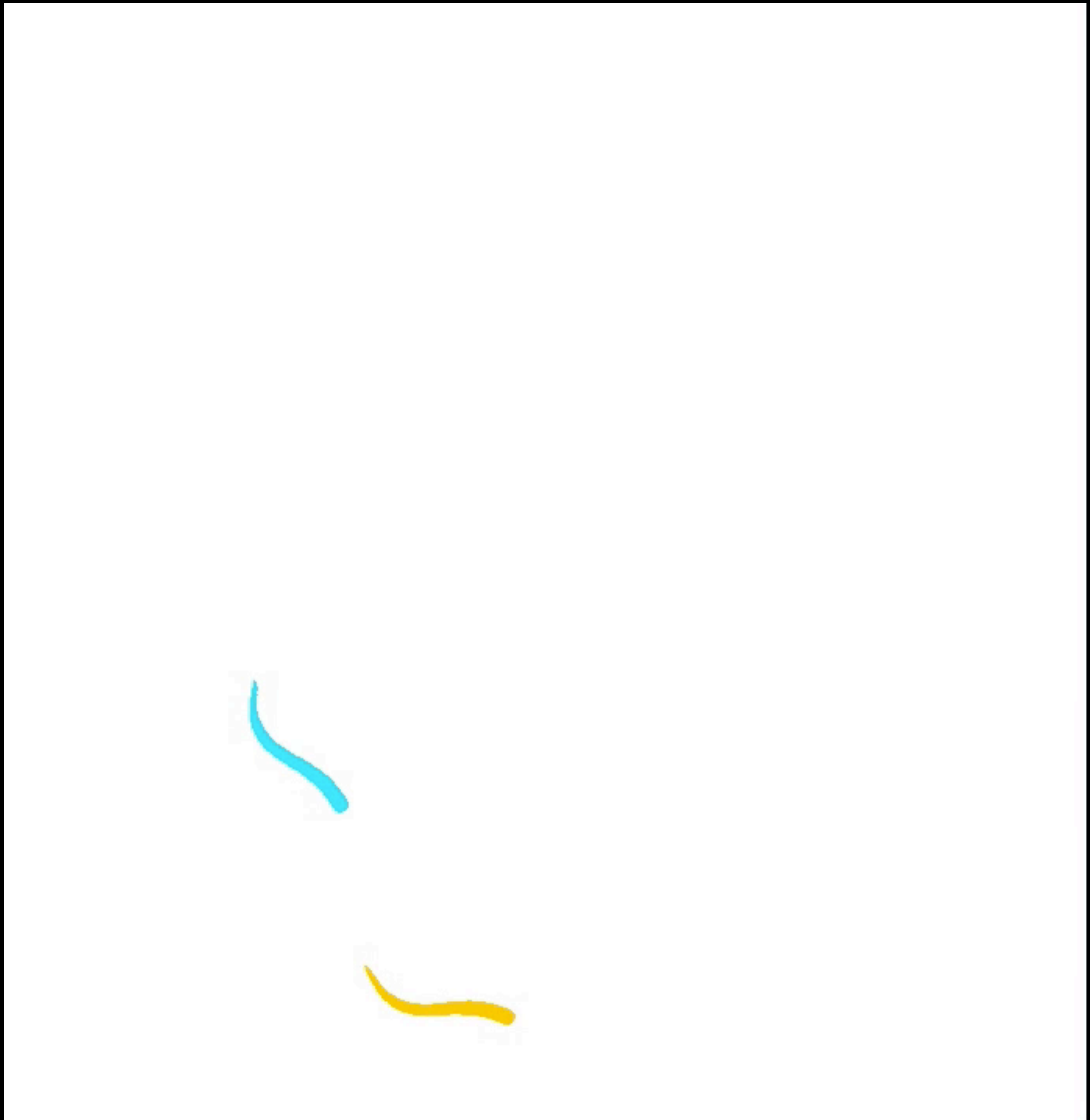


co-contractions

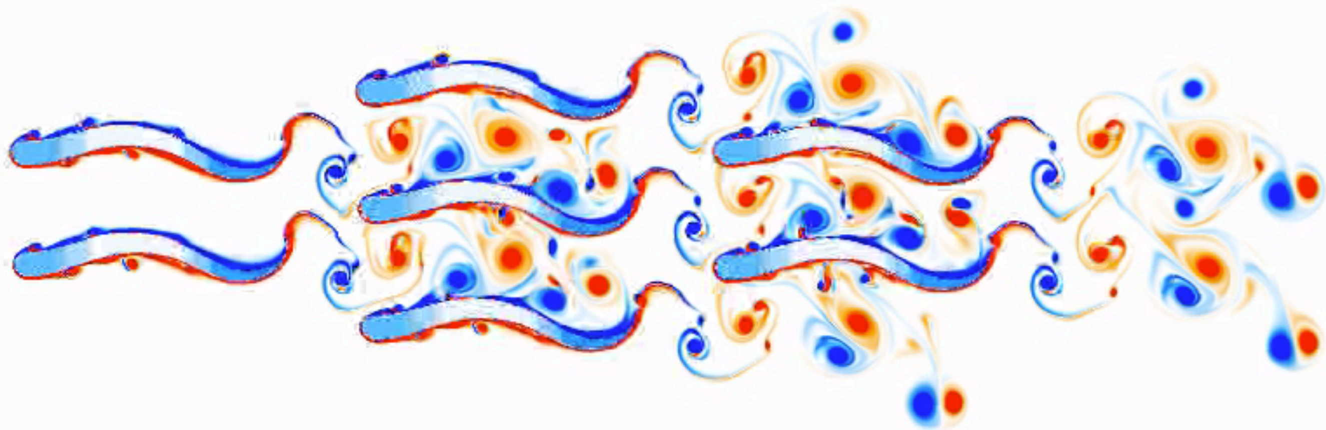
Swimming with & without co-contractions



Turning with and without co-contractions

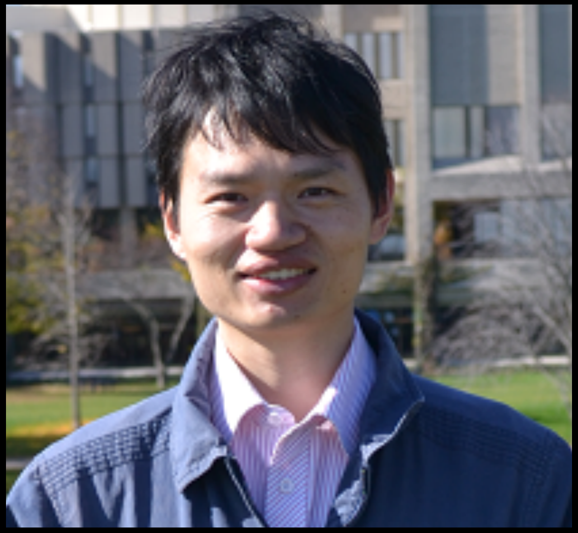


Fish school



Acknowledgements

Amneet P.S.
Bhalla



Wenjun
Kou

Boyce
Griffith

NSF

NUIT's Quest

IBAMR

https://github.com/IBAMR/IBAMR