Current and Planned AMR Support in PETSc

Toby Isaac
tisaac@ices.utexas.edu

The University of Chicago at Austin

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motivation

why adapt?

1. Your non-adaptive calculations have reached the end of your resources (or the end of weak-scalability), and you want to push back.
2. You have a performance model that predicts it can help.

hp-FEM theory

predicts exponential convergence in N_{dof}:

- If we want zero error, it's worth it.
- If we have a nonzero tolerance, we must consider that hp systems require more resources per dof to solve than uniform, low-order systems. There is always a crossover.

wishlist

data structures

- a good interface for the user to specify local mesh sizes
  - " " " " " " " " " local metric tensors
- composable mesh formats
- configurability

numerical methods

- *a posteriori* mesh refinement and coarsening

The latter requires the former. The former clearly lives in DM, but the latter doesn't fit anywhere at the moment:

- KSP, SNES, TS, TAO: the dimension is fixed.
- PetscQoI_AMR (hypothetical): compile years of research into *a posteriori* methods.

mesh formats

DA

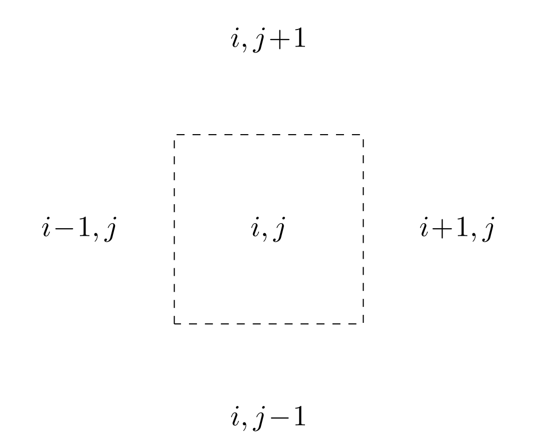
Plex

Performance ⟷ Generality

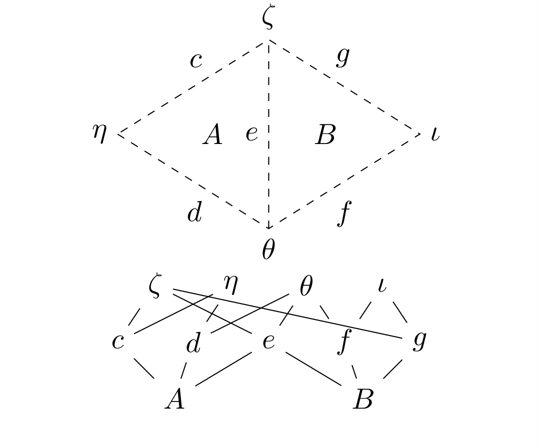
mesh formats

Who is my neighbor?

DA



Plex



mesh formats

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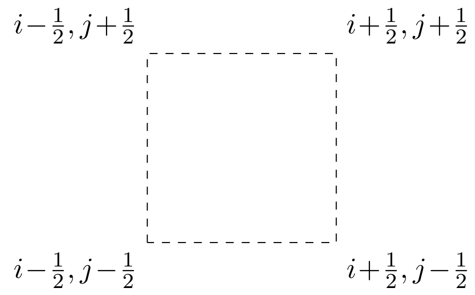
$i-\frac{1}{2},j+\frac{1}{2}$
$i+\frac{1}{2},j+\frac{1}{2}$
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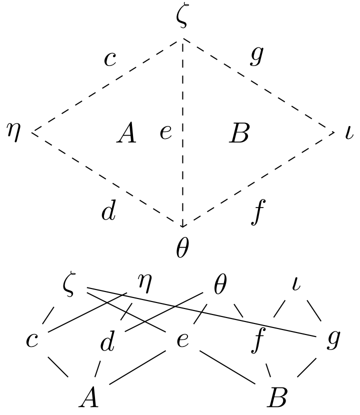
$i+\frac{1}{2},j-\frac{1}{2}$

Plex



mesh formats

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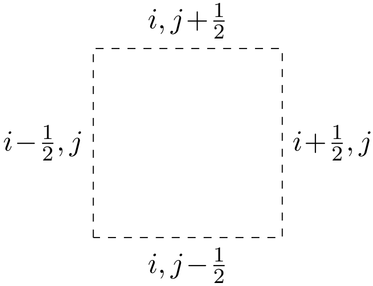

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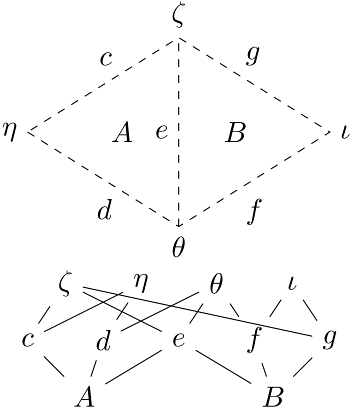

mesh formats

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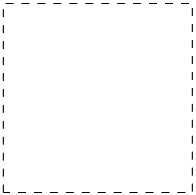

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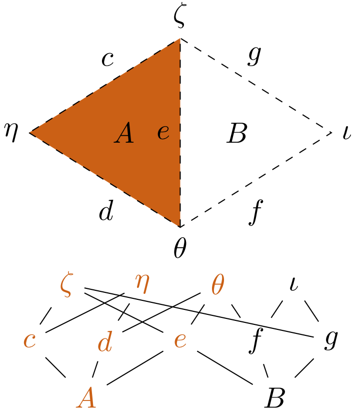

mesh formats

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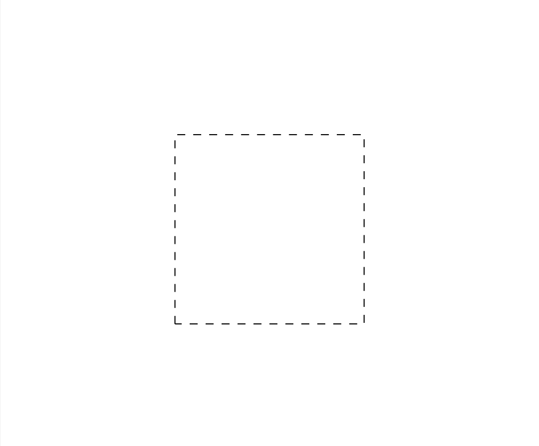

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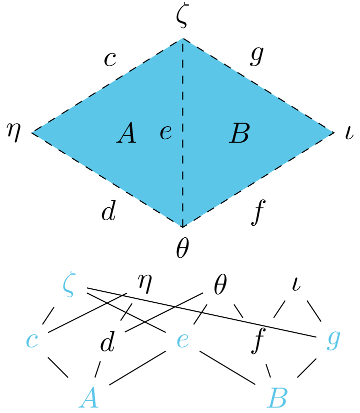

mesh formats

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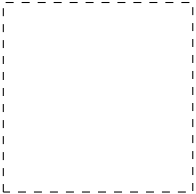
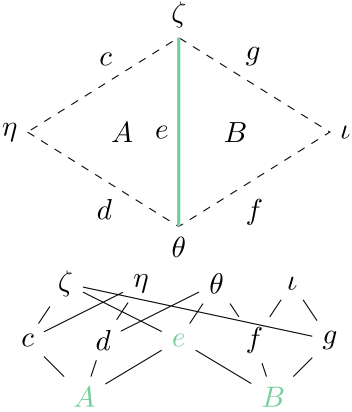


Plex

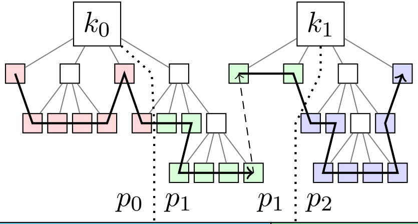


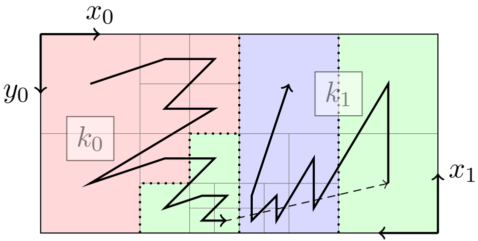
p4est

Overview

The p4est library (p4est.org, lead developer Carsten Burstedde) provides scalable AMR routines via a *forest-of-octrees/quadtrees*:

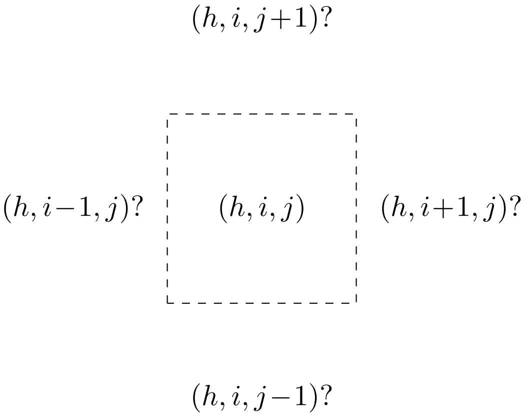
- a unstructured hexahedral mesh (“the forest”);
- where each hexahedron contains an arbitrarily refined octree;
- space-filling curve (SFC) orders elements;
- philosophy: as-simple-as-possible coarse mesh describes geometry, refinement captures all detail.





p4est

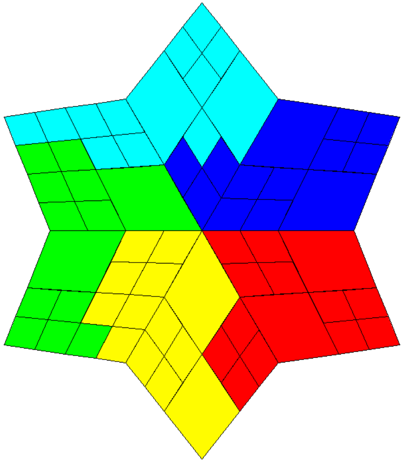
p4est: linear octree (pointerless octants)



$(h, i, j+1)?$

$(h, i-1, j)?$ (h, i, j) $(h, i+1, j)?$

$(h, i, j-1)?$



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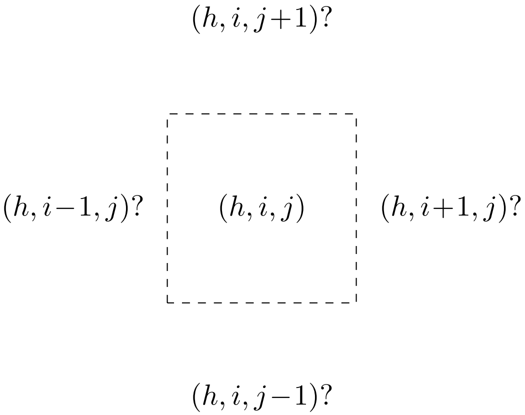
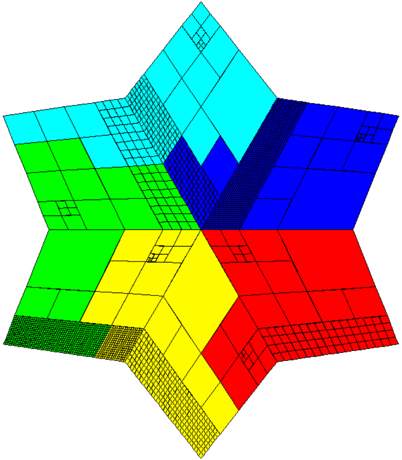


$(h, i, j{+}1)?$

$(h, i{-}1, j)?$ (h, i, j) $(h, i{+}1, j)?$

$(h, i, j{-}1)?$



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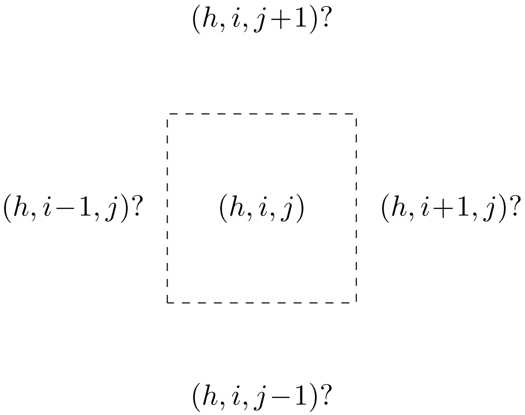

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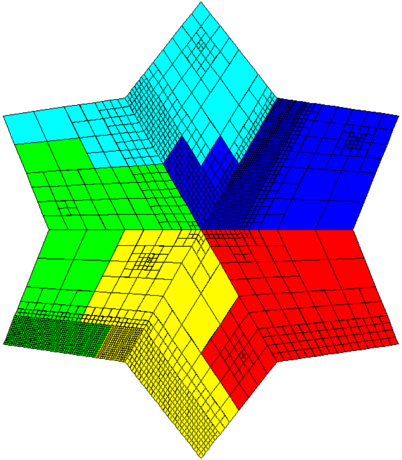

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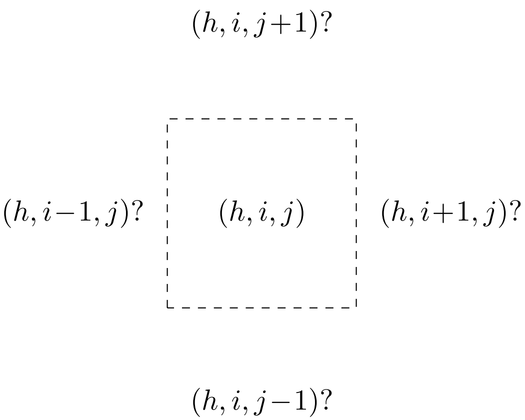

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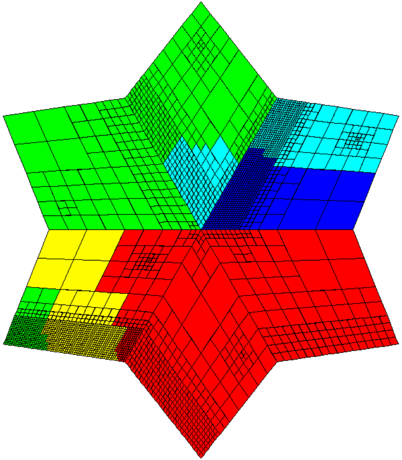
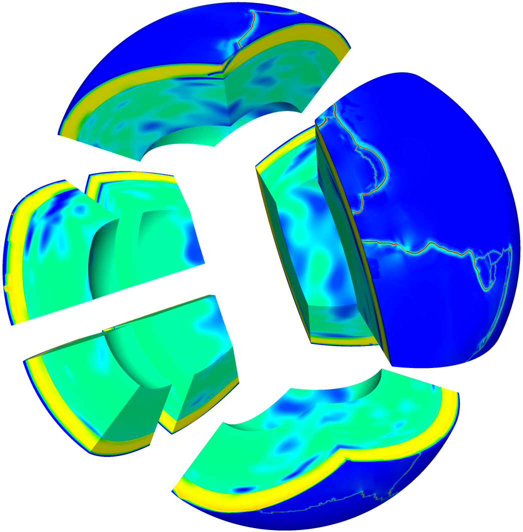


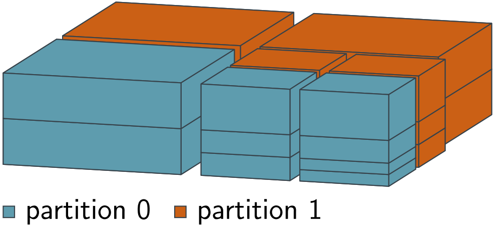


Mantle convection from (Stadler et al., 2010).

wanting an interface to solvers

Extension for thin domains (ice sheets)

(See my CSE15 slides, or arXiv:1406.6573)



GMG+AMG

(Sundar et al., SC12), (Rudi et al., best poster SC14, Rudi et al., paper submitted SC15): geometric-algebraic multigrid for mantle convection built on p4est.

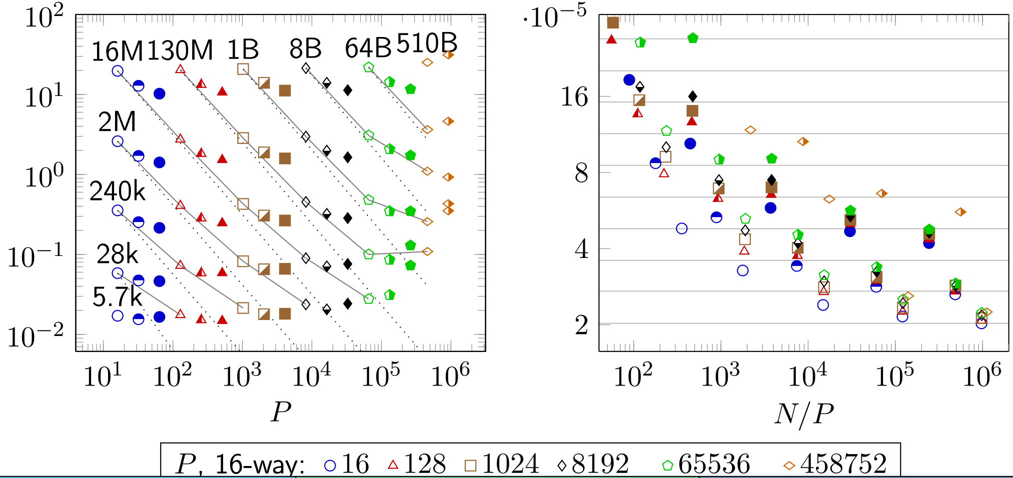
The implementation of GMG+AMG is very intrusive: it would've required more time than I had to adapt it to the thin-domain extension.

wanting extensibility, configurability

Scalability to 458K BG/Q cores of JUQUEEN from arXiv:1406.0089 (accepted SISC 2015).

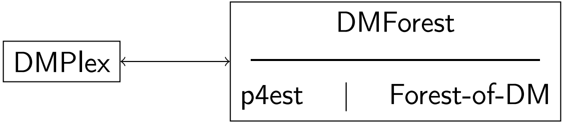
forest-to-mesh runtime in seconds

forest-to-mesh runtime in secs./(N/P)



DMForest

the plan



1. Support non-conforming cell interfaces in DMPlex. [done, in PETSc 3.6]
2. p4est-to-DMPlex conversion [done, in devel p4est, needs trivial update for PETSc 3.6]
3. DMForest interface, with p4est as first implementation. [stub]
   - Immediate solver support via backend conversion to DMPlex.
   - High-performance, native solver support if needed.
4. Runtime conversion of DMPlex to root forest (`-plex_convert_to_forest -forest_type X`):
   - user specifies coarse DMPlex that captures topology/geometry/X, has immediate access to all DMForest implementations.

Forest-of-DM

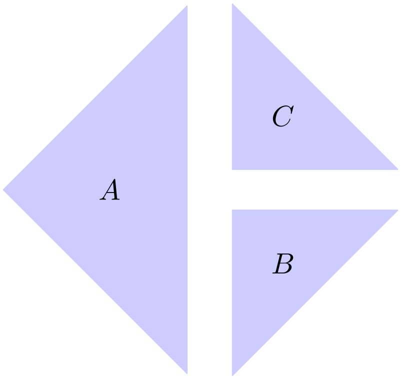
Design principle

Leaves on one level are roots on the next.

- All other aspects (coarse-mesh redundancy, partitioning, implicit vs. explicit tree) are implementation specifics.
- Other SFC approaches: Hilbert, Peano, Sierpinski
- Other external packages
- In particular, Forest-of-DA:
  - would allow a fair assessment of adaptivity overhead on domains other than boxes, i.e., compare locally-refined p4est to DA rather than to uniformly-refined p4est.

non-conforming cells in DMPlex

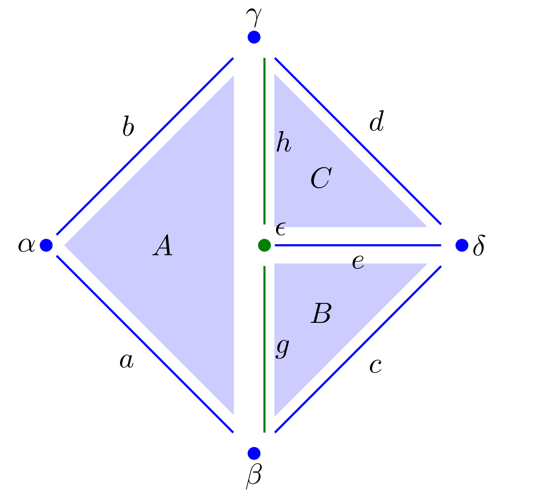
What is the proper completion of the DMPlex?



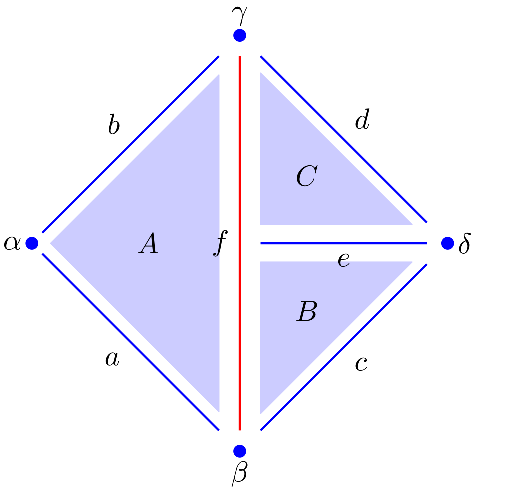
- We want contributors to write contributions meant for conforming meshes that still work for non-conforming meshes.

which is the proper partition completion?

FVM

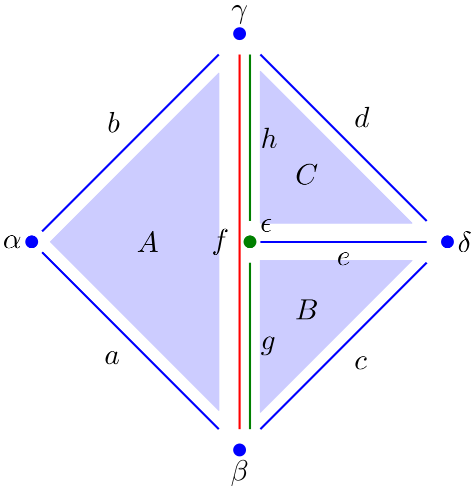


FEM



We add a tree structure to represent hierarchical points

DMPlex with hierarchical points



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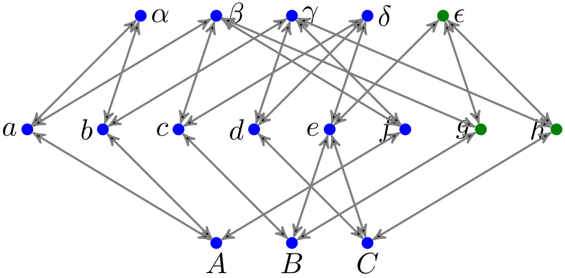
DMPlex with hierarchical points



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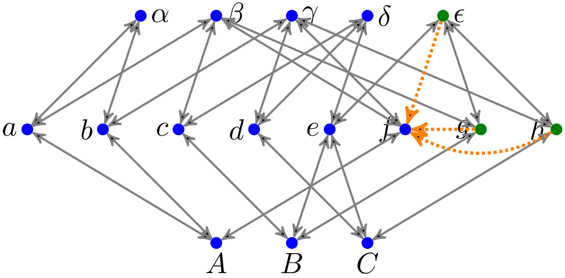
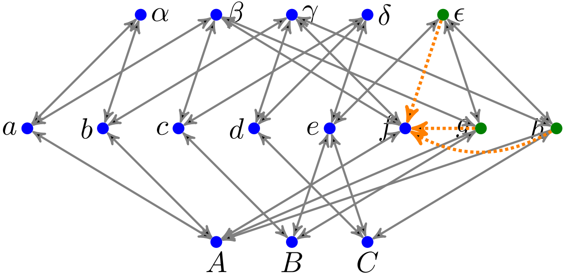
DMPlex with hierarchical points



DMPlexSetTree(...)

We add a tree structure to represent hierarchical points

DMPlex with hierarchical points



DMPlexSetTree(...)

- Supports automatically recalculated so adjacencies can be computed.
- Compatible with Michael Lange's mesh redistribution work.

We add a tree structure to represent hierarchical points

the two rules for FEM and FVM calculations

1. dofs on roots:
   - correct global sections automatically calculated.
2. quadrature on leaves:
   - not an issue for FEM if cells don't overlap.
   - FVM (and DG): only leaves in facet loops.

constraints

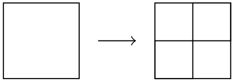
- Child points don't have global dofs (i.e., in global vectors), but they do have dofs in local vectors:
  - so that DMPlexVecGet/SetClosure() [get/set the closure of a function on a cell] always works.
- "Hanging-node" constraints must enforce $H^1/H^{\mathsf{curl}}/H^{\mathsf{div}}$ continuity, i.e., there is an interpolation operator I_a^c from *anchor* dofs to *constrained* dofs that must be applied.
  - Incorporated into DMGlobalToLocal()/DMLocalToGlobal(): constraints are always satisfied on local vectors/summed properly into global residuals.
  - DMPlexMatSetClosure() constructs the correct Jacobian.

automatic constraints

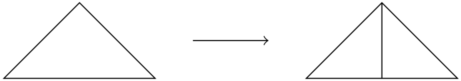
computing I_a^c

Most non-conforming meshes (such as p4est meshes) come from "reference" refinement rules:

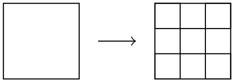
Uniform $1/2^d$ (Hilbert/Morton)



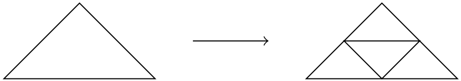
Bisection (Sierpinski)



Uniform $1/3^d$ (Peano)



Uniform $1/2^d$



- DMPlexSetReferenceTree(...):
  - (PetscFE) Hanging constraints computed on the reference refinement, copied into I_a^c (works even for curvilinear meshes).
  - Structured anisotropic refinement fits in this framework as well.

summary & current work

- DMForest will be a bridge from the successful, scalable p4est library to an extensible structured AMR interface:
  - Increase the availability of adaptivity to non-experts.
  - Develop (and fairly test) algorithms for $> 10^6$ processes.
- DMPlex can now serve as a general format for (hierarchically) non-conforming meshes, as it already has for conforming meshes.
  - Finite element support is complete (as complete as PetscFE).
  - Finite volume and discontinuous Galerking methods are in development, but the interface can be improved to hide complexity from the user.

Thank you!