The Portable Extensible Toolkit for Scientific Computing

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Outline

1. What can PETSc do?
   - What is PETSc?
   - Who uses PETSc?
2. What's New in PETSc?
3. Conclusion

Outline

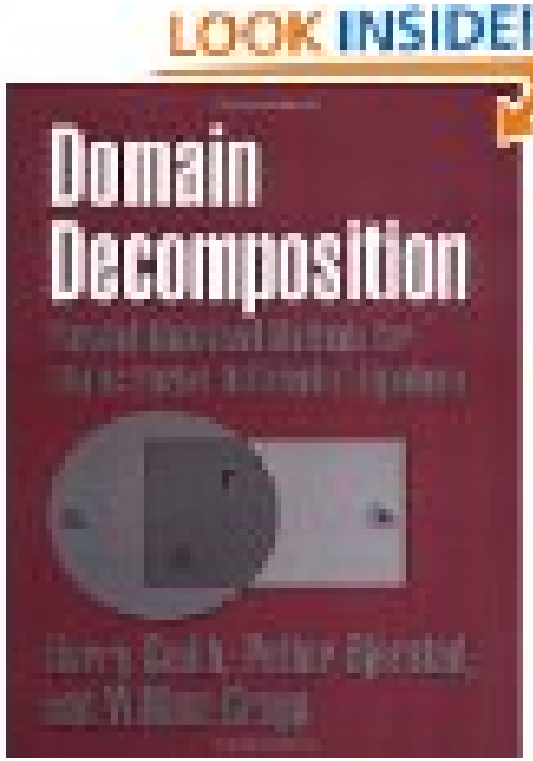
1. What can PETSc do?
   - What is PETSc?
   - Who uses PETSc?

How did PETSc Originate?

PETSc was developed as a Platform for Experimentation

We want to experiment with different

- Models
- Discretizations
- Solvers
- Algorithms
  - which blur these boundaries



The Role of PETSc

Developing parallel, nontrivial PDE solvers that deliver high performance is still difficult and requires months (or even years) of concentrated effort.

PETSc is a toolkit that can ease these difficulties and reduce the development time, but it is not a black-box PDE solver, nor a silver bullet.

— Barry Smith

What is PETSc?

A freely available and supported research code

- Download from http://www.mcs.anl.gov/petsc
- Free for everyone, including industrial users
- Hyperlinked manual, examples, and manual pages for all routines
- Hundreds of tutorial-style examples
- Support via email: petsc-maint@mcs.anl.gov
- Usable from C, C++, Fortran 77/90, and Python

What is PETSc?

- Portable to any parallel system supporting MPI, including:
  - Tightly coupled systems
    - Cray XT5, BG/P, NVIDIA Tesla, Earth Simulator, Sun Blade
  - Loosely coupled systems, such as networks of workstations
    - IBM, Mac, Sun, PCs running Linux or Windows
- PETSc History
  - Begun September 1991
  - Over 60,000 downloads since 1995 (version 2)
  - Currently 400 per month
- PETSc Funding and Support
  - Department of Energy
    - SciDAC, MICS Program, INL Reactor Program
  - National Science Foundation
    - CIG, CISE, Multidisciplinary Challenge Program

The PETSc Team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Barry Smith


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Jed Brown


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Hong Zhang


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Dmitry Karpeev

Outline

1. What can PETSc do?
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Who Uses PETSc?

- Computational Scientists
  - PyLith (CIG), Underworld (Monash), Magma Dynamics (LDEO, Columbia), PFLOTRAN (DOE)
- Algorithm Developers
  - Iterative methods and Preconditioning researchers
- Package Developers
  - SLEPc, TAO, DealII, PETSc-FEM, MagPar, PetFMM, PetRBF

What Can We Handle?

- PETSc has run implicit problems with over 500 billion unknowns
  - UNIC on BG/P and XT5
  - PFLOTRAN for flow in porous media
- PETSc has run on over 224,000 cores efficiently
  - UNIC on the IBM BG/P Intrepid at ANL
  - PFLOTRAN on the Cray XT5 Jaguar at ORNL
- PETSc applications have run at 22 Teraflops
  - Kaushik on XT5
  - LANL PFLOTRAN code

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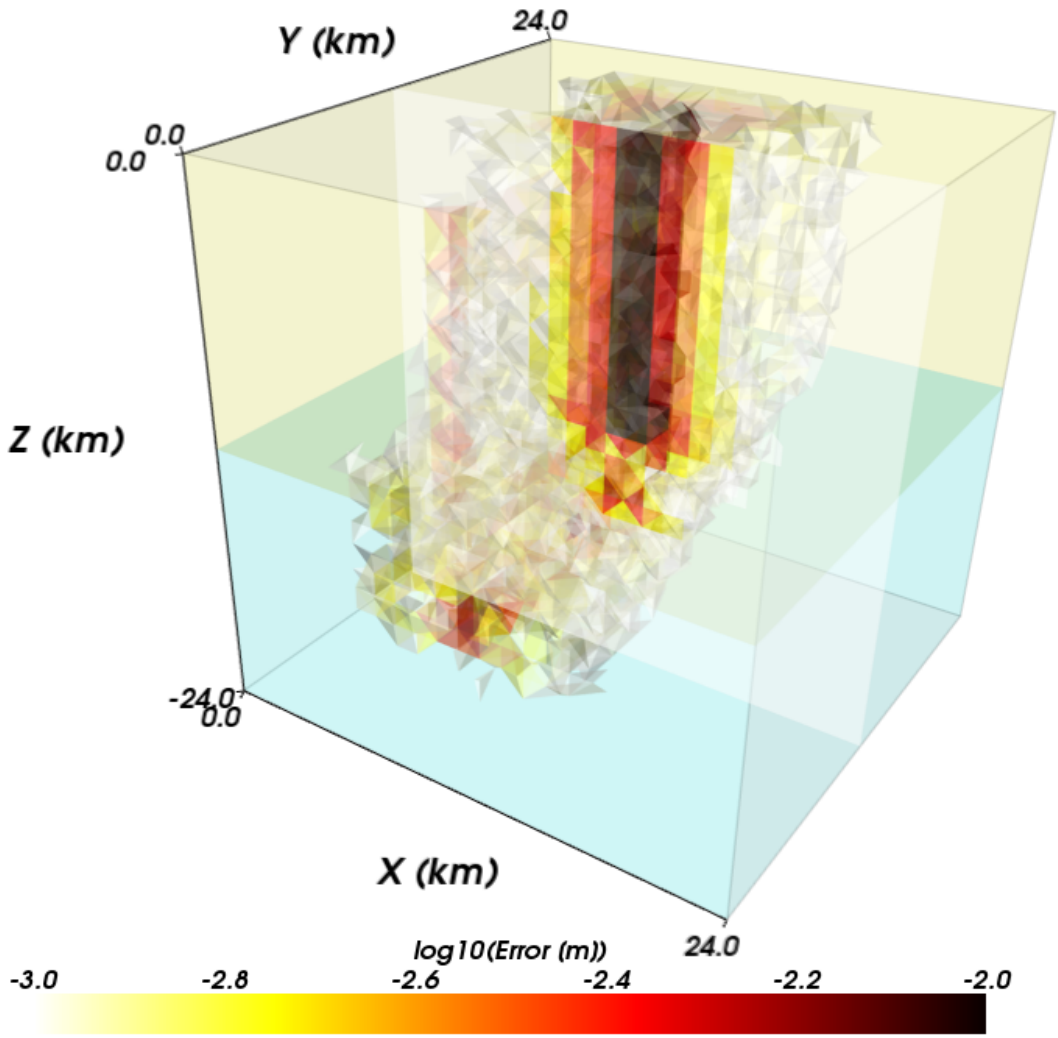
PyLith

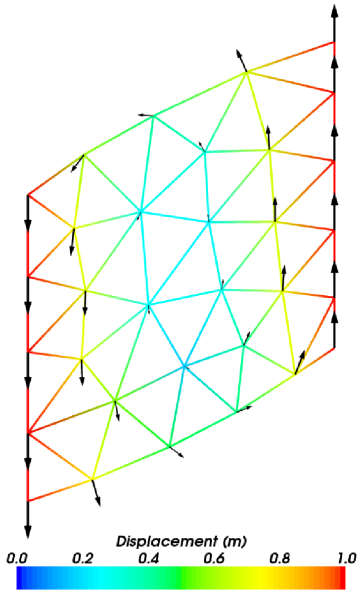
- Multiple problems
  - Dynamic rupture
  - Quasi-static relaxation
- Multiple models
  - Nonlinear visco-plastic
  - Finite deformation
  - Fault constitutive models
- Multiple meshes
  - 1D, 2D, 3D
  - Hex and tet meshes
- Parallel
  - PETSc solvers
  - Sieve mesh management

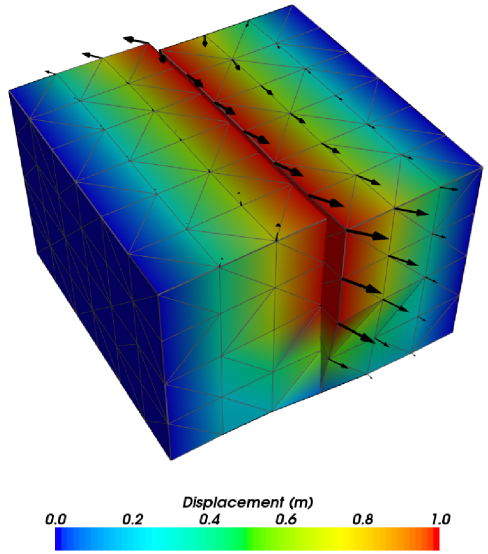


[a] Aagaard, Knepley, Williams

Multiple Mesh Types

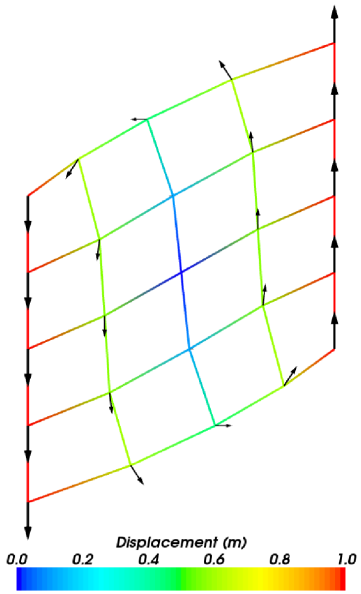
Triangular

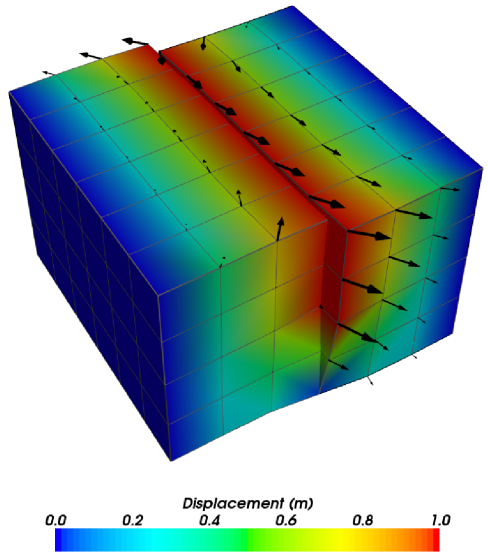




Tetrahedral

Rectangular

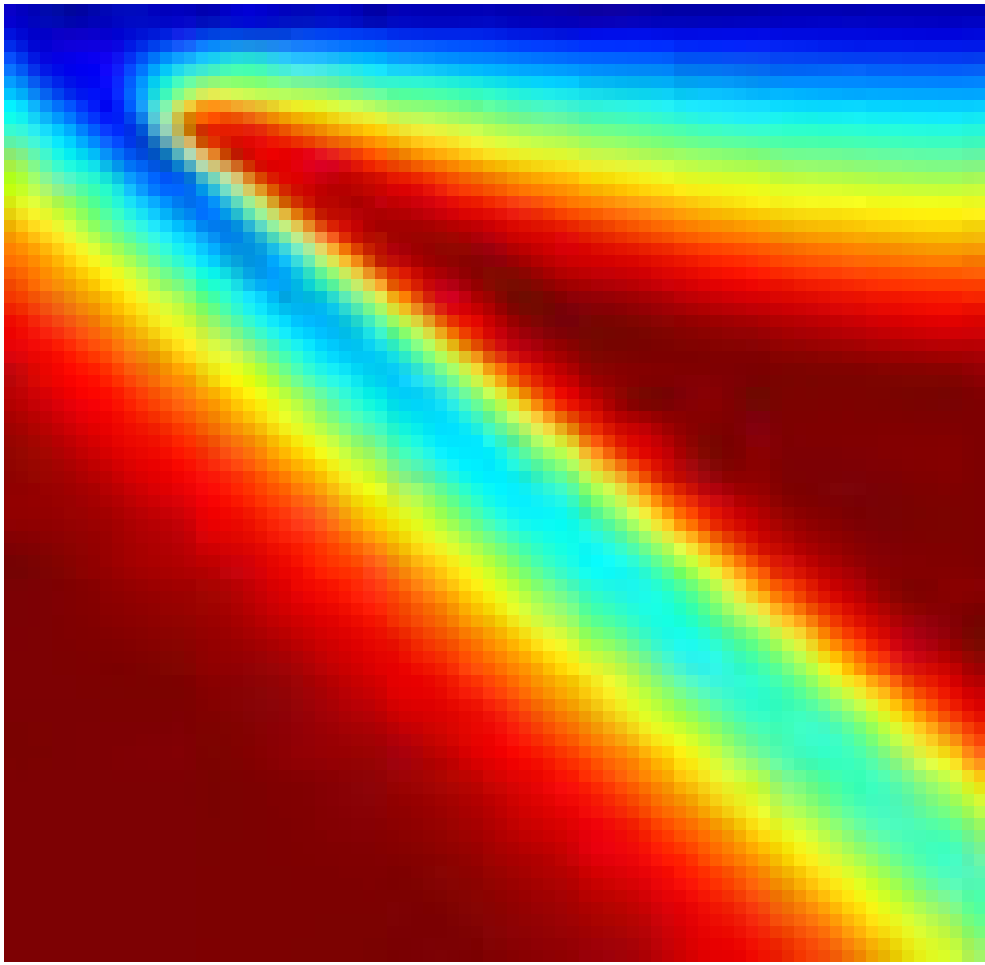




Hexahedral

Magma Dynamics

- Couples scales
  - Subduction
  - Magma Migration
- Physics
  - Incompressible fluid
  - Porous solid
  - Variable porosity
- Deforming matrix
  - Compaction pressure
- Code generation
  - FEniCS
- Multiphysics Preconditioning
  - PETSc FieldSplit



[a]Katz, Speigelman

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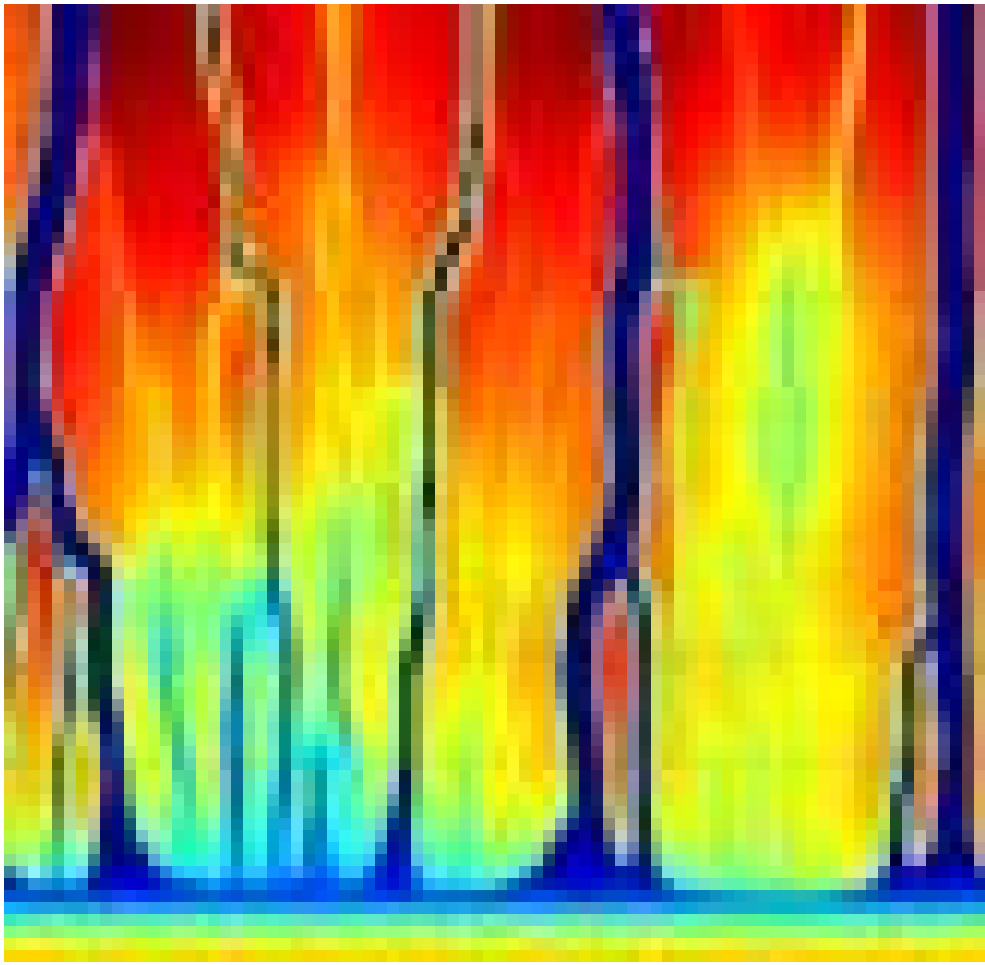
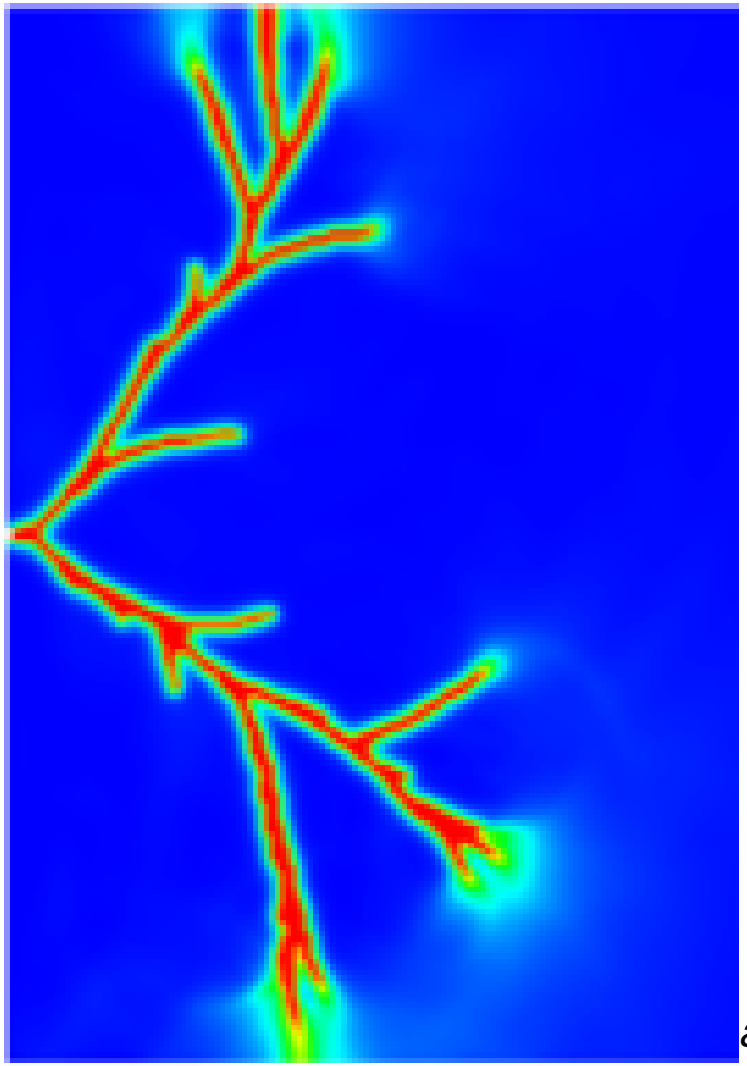


[a]Katz, Speigelman

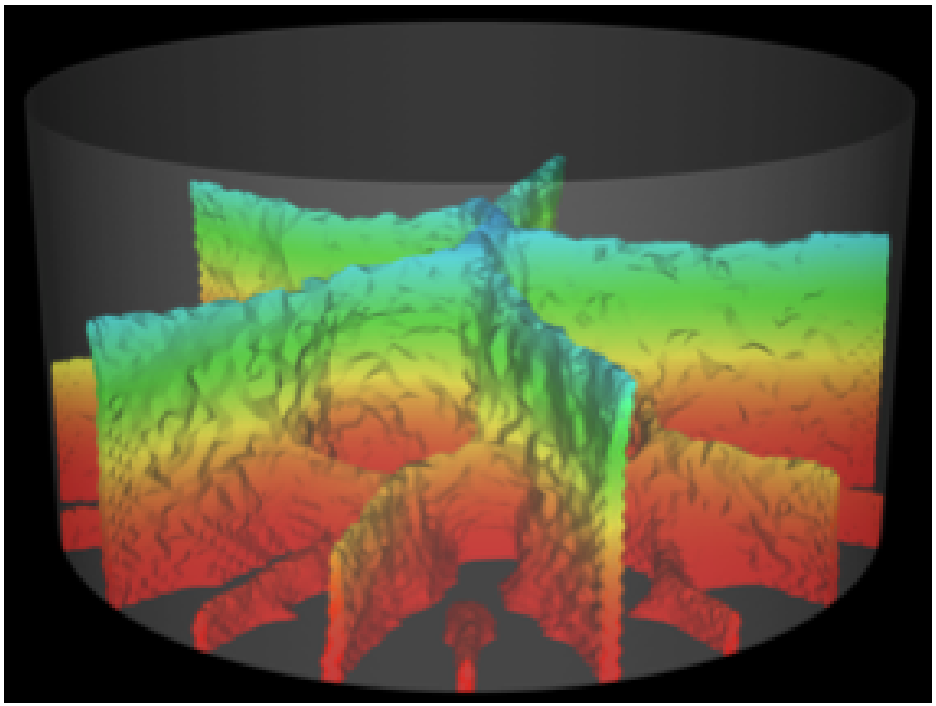
Fracture Mechanics

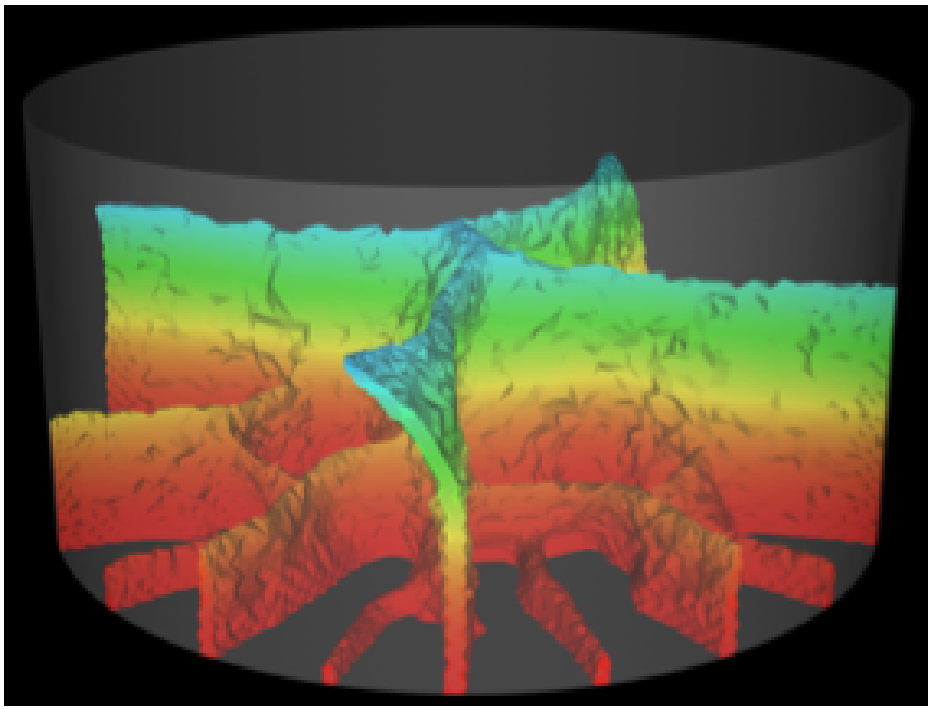
- Full variational formulation
  - Phase field
  - Linear or Quadratic penalty
- Uses TAO optimization
  - Necessary for linear penalty
  - Backtacking
- No prescribed cracks
  - Arbitrary crack geometry
  - Arbitrary intersections
- Multiple materials
  - Composite toughness



[a] Bourdin

Fracture Mechanics





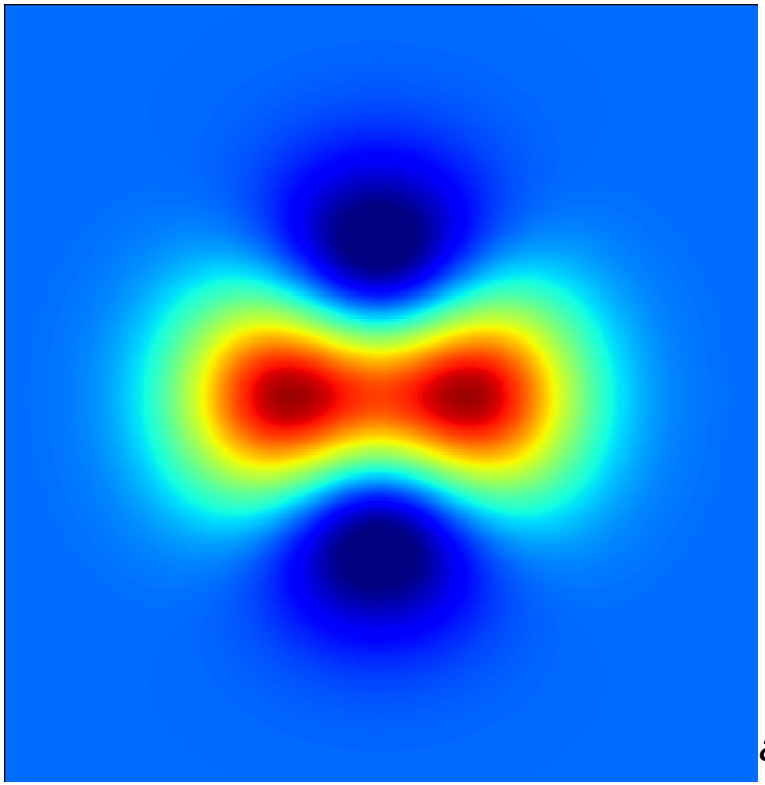
[1]

[1] Bourdin

Vortex Method

t = 000

- Incompressible Flow
  - Gaussian vortex blobs
  - High Re
- PetFMM
  - 2D/3D domains
  - Automatic load balancing
  - Variety of kernels
  - Optimized with templates
- PetRBF
  - Variety of RBFs
  - Uses PETSc solvers
  - Scalable preconditioner
- Parallelism
  - MPI
  - GPU



[a]Cruz, Yokota, Barba, Knepley

Vortex Method

t = 100

- Incompressible Flow
  - Gaussian vortex blobs
  - High Re
- PetFMM
  - 2D/3D domains
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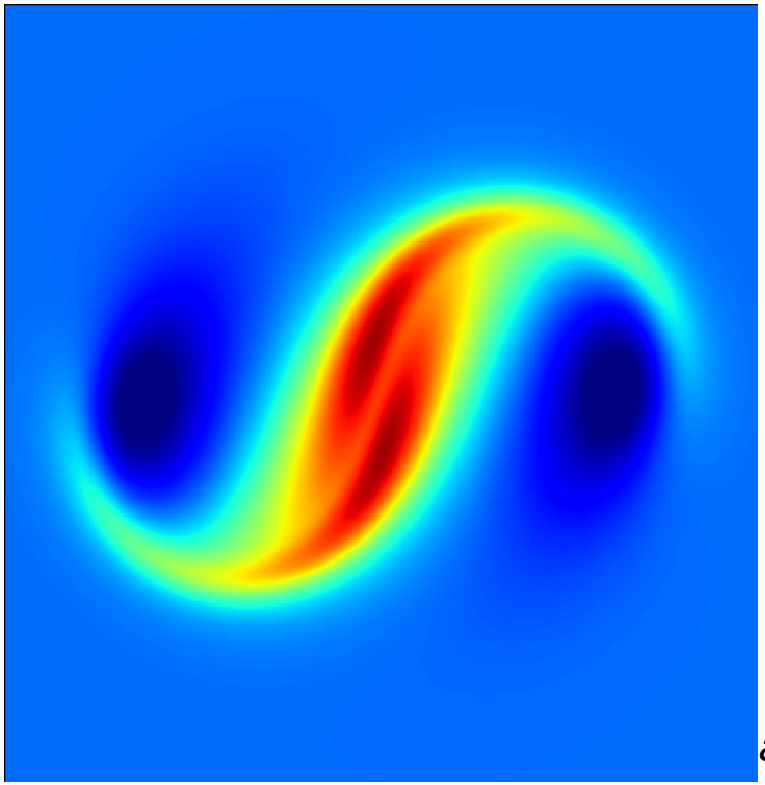

[a]Cruz, Yokota, Barba, Knepley

Vortex Method

t = 200

- Incompressible Flow
  - Gaussian vortex blobs
  - High Re
- PetFMM
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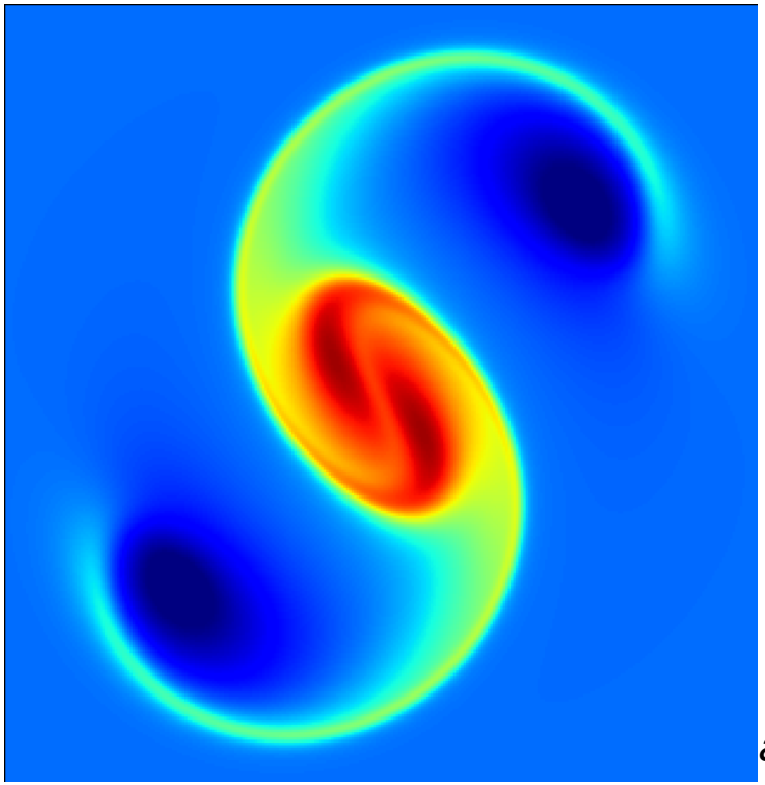

[a]Cruz, Yokota, Barba, Knepley

Vortex Method

t = 300

- Incompressible Flow
  - Gaussian vortex blobs
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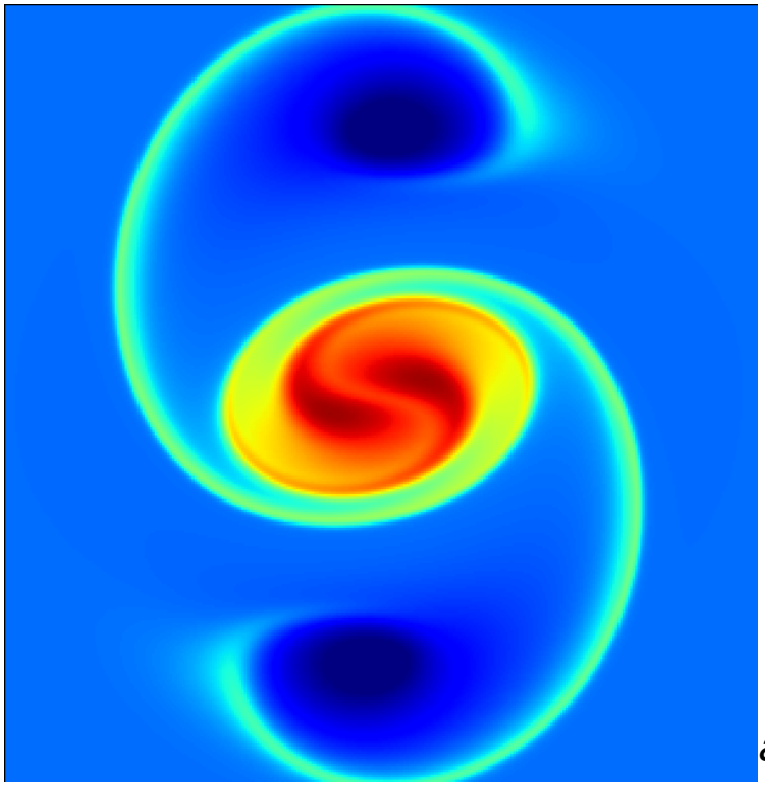

[a]Cruz, Yokota, Barba, Knepley

Vortex Method

t = 400

- Incompressible Flow
  - Gaussian vortex blobs
  - High Re
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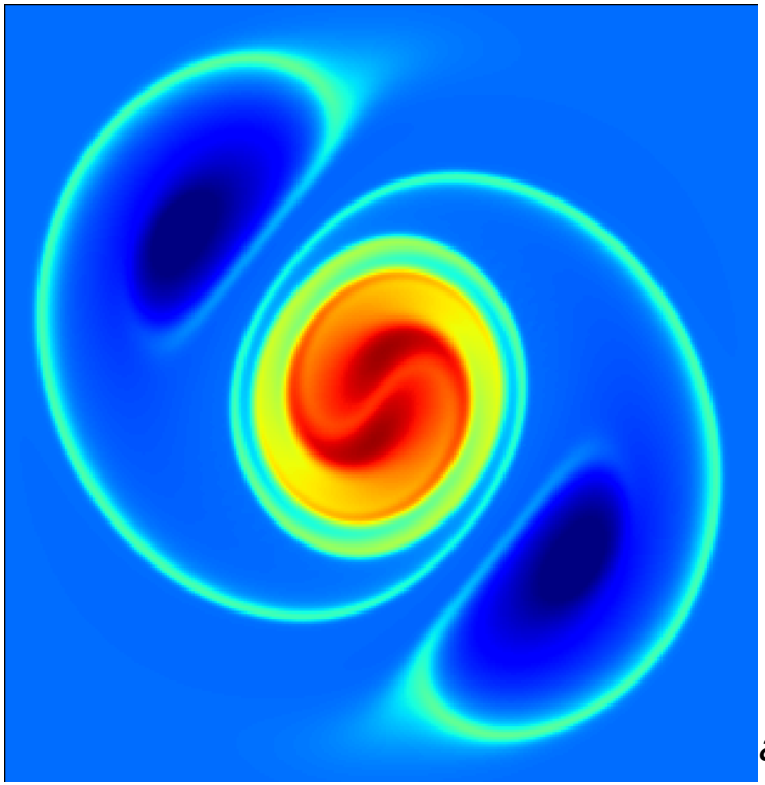

[a]Cruz, Yokota, Barba, Knepley

Vortex Method

t = 500

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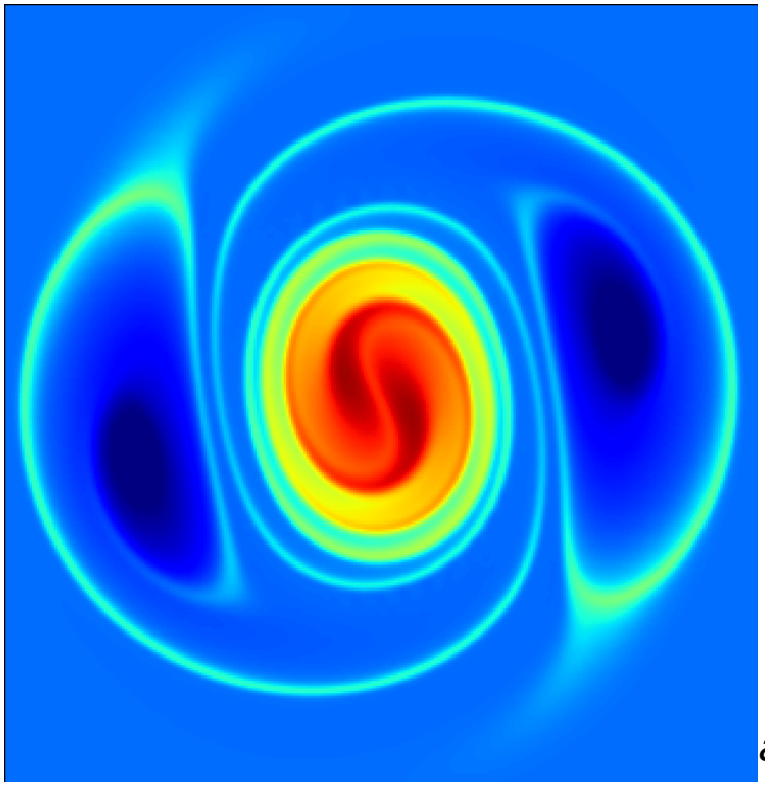

[a]Cruz, Yokota, Barba, Knepley

Vortex Method

t = 600

- Incompressible Flow
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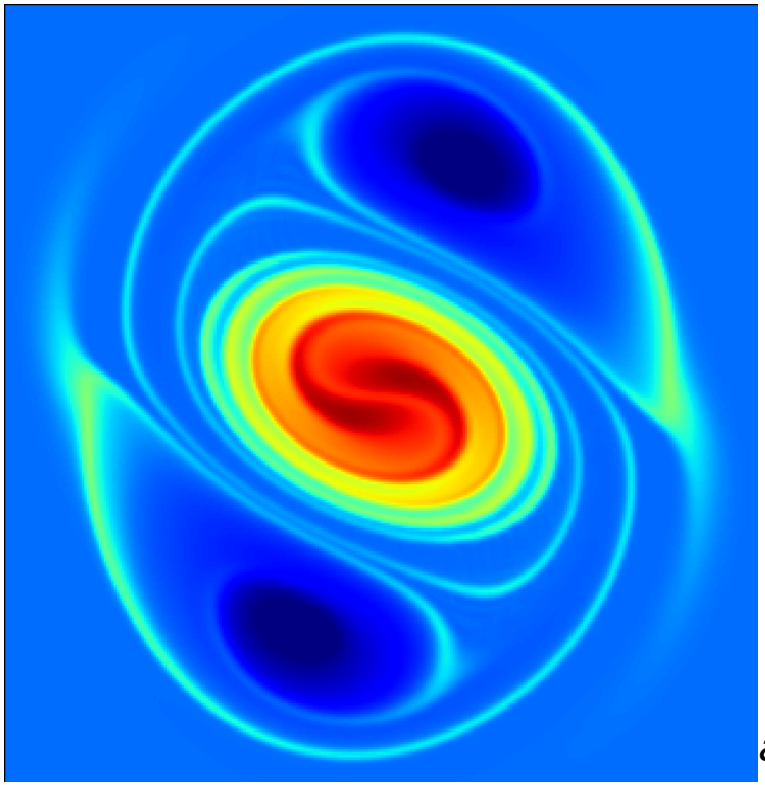

[a]Cruz, Yokota, Barba, Knepley

Vortex Method

t = 700

- Incompressible Flow
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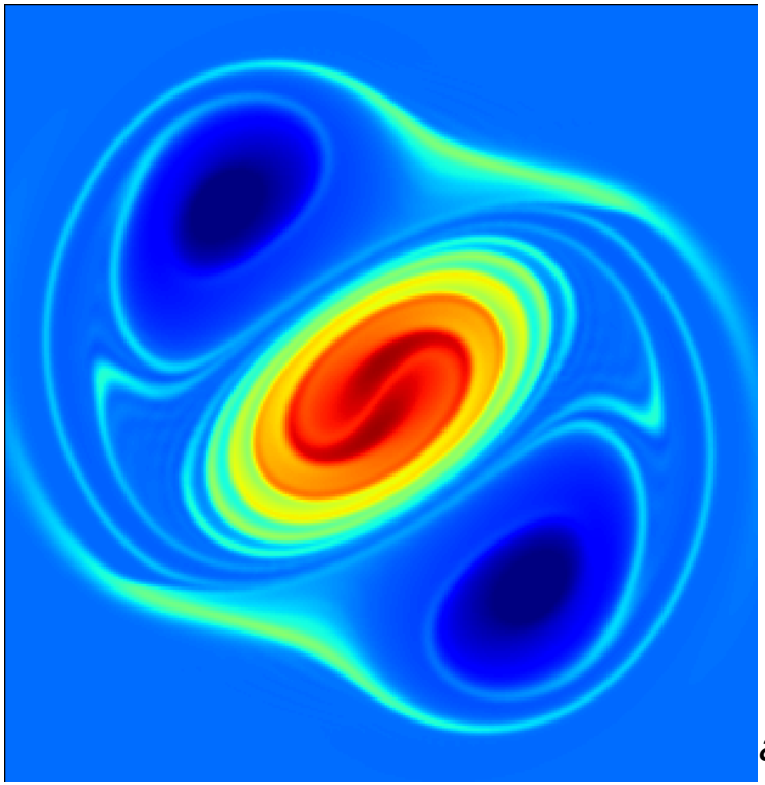

[a]Cruz, Yokota, Barba, Knepley

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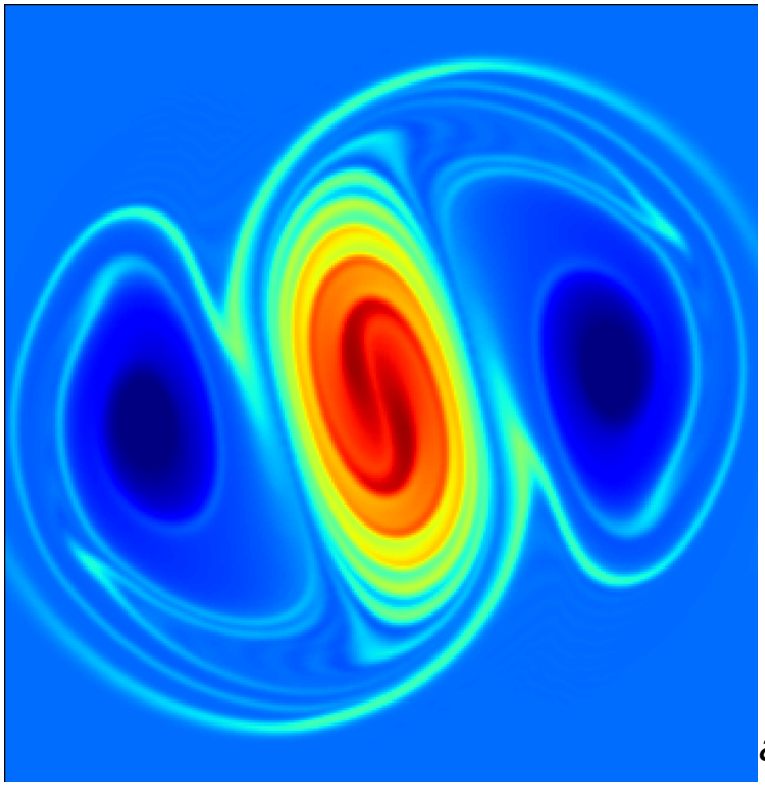
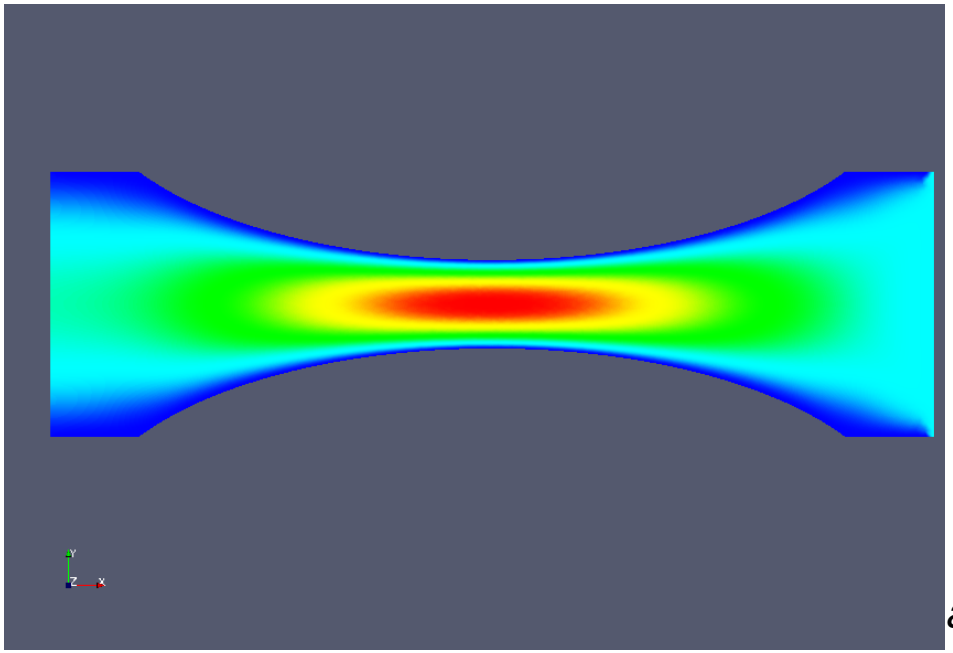


[a]Cruz, Yokota, Barba, Knepley

FEniCS-Apps
Rheagen

- Rheologies
  - Maxwell
  - Grade 2
  - Oldroyd-B
- Stabilization
  - DG
  - SUPG
  - EVSS
  - DEVSS
  - Macroelement
- Automation
  - FIAT (elements)
  - FFC (weak forms)



[a]

[a] Terrel

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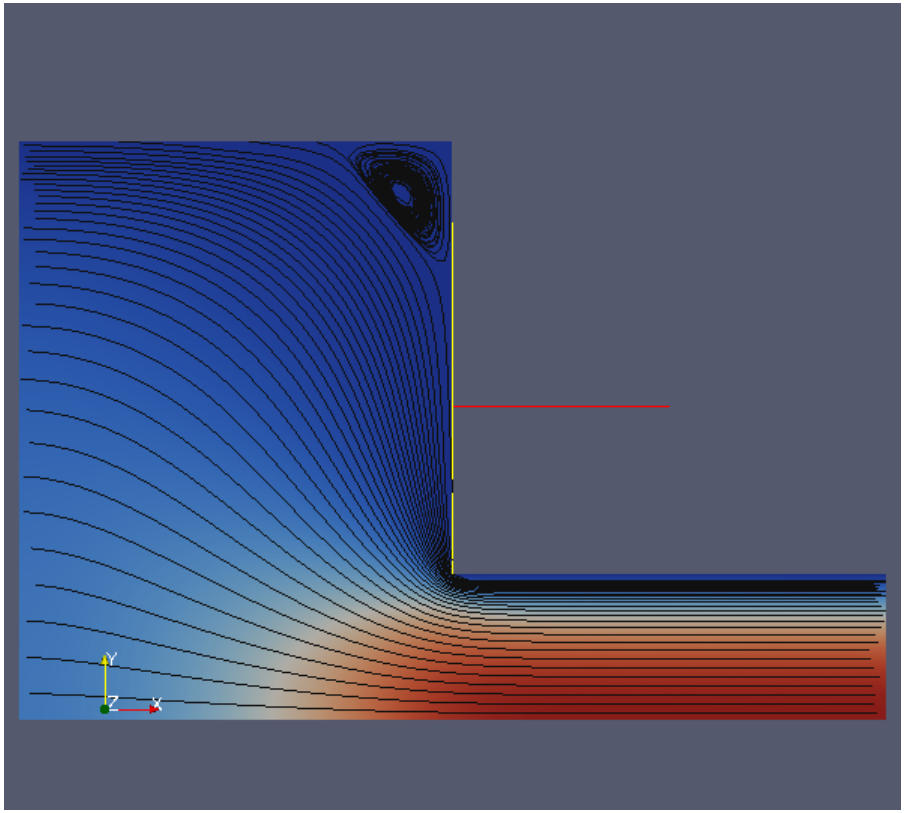

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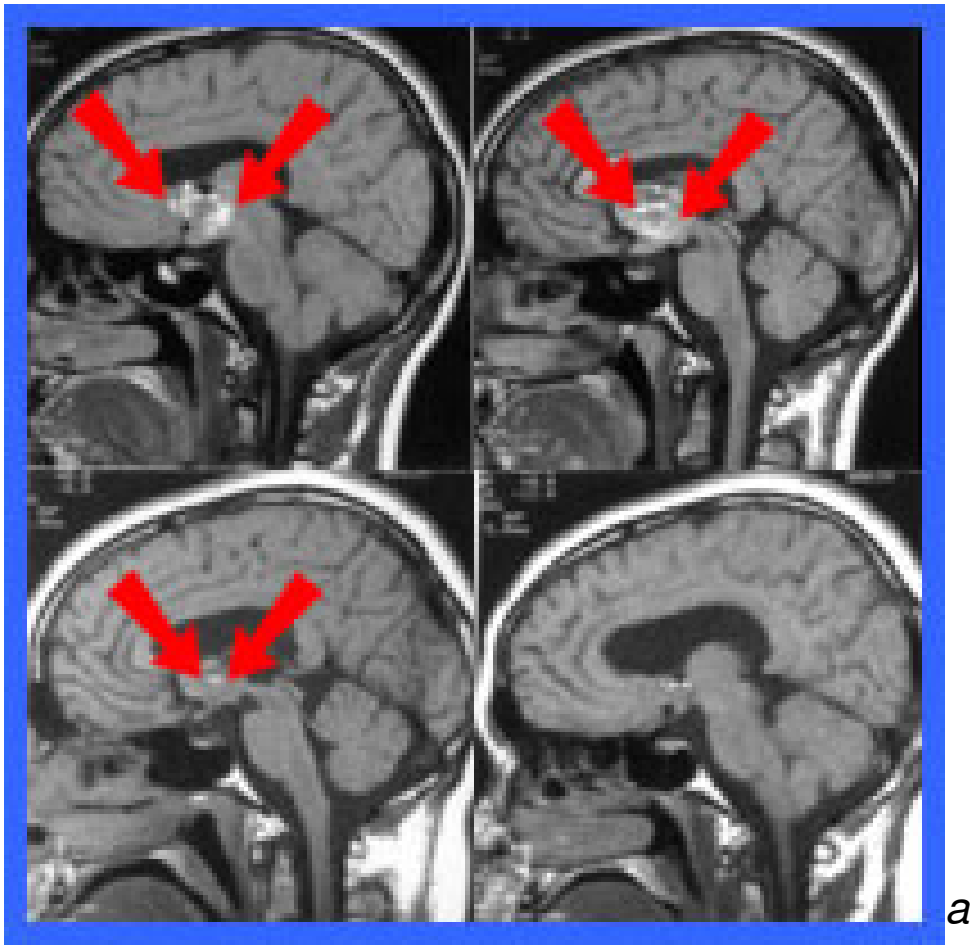
[a] Terrel

Real-time Surgery

- Brain Surgery
  - Elastic deformation
  - Overlaid on MRI
  - Guides surgeon
- Laser Thermal Therapy
  - PDE constrained optimization
  - Per-patient calibration
  - Thermal inverse problem



[a]

[a] Warfield, Ferrant, et.al.

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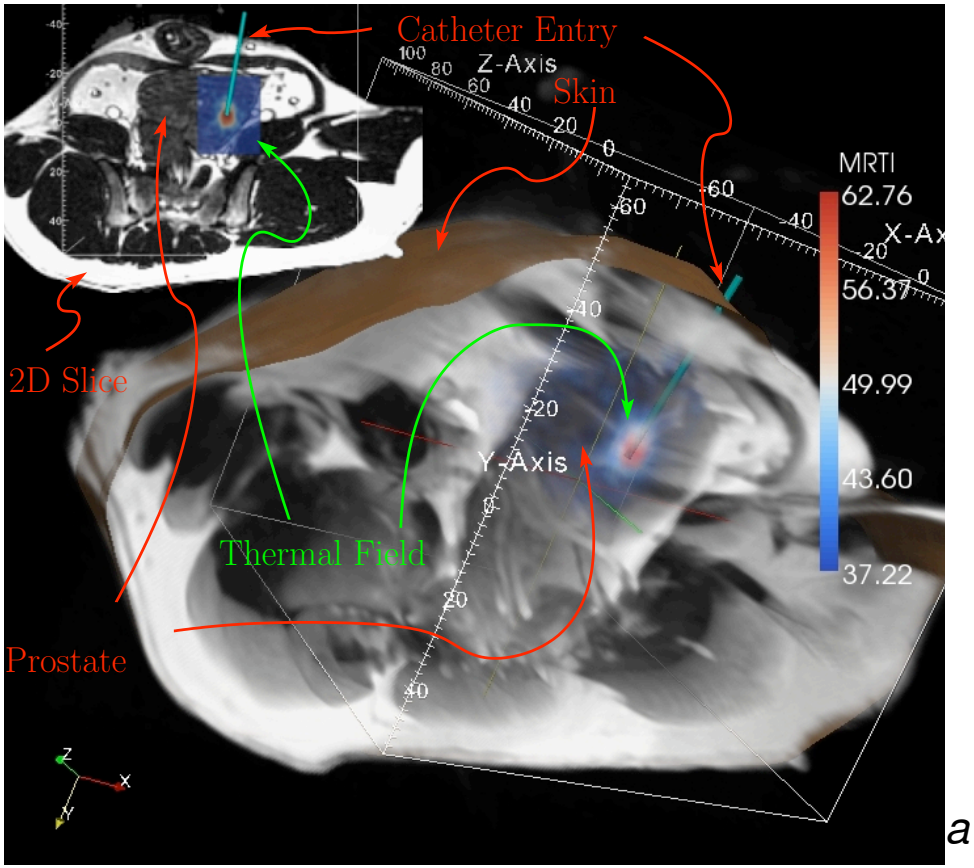

[a]

[a]Fuentes, Oden, et.al.

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1 What can PETSc do?

2 What's New in PETSc?
- Python Bindings
- Physics-Based Preconditioning
- PETSc-GPU
- FEM

3 Conclusion

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2 What's New in PETSc?
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numpy

numpy is ideal for building Python data structures

- Supports multidimensional arrays
- Easily interfaces with C/C++ and Fortran
- High performance BLAS/LAPACK and functional operations
- Python 2 and 3 compatible
- Used by petsc4py to talk to PETSc

petsc4py

petcs4py provides Python bindings for PETSc

- Provides ALL PETSc functionality in a Pythonic way
  - Logging using the Python `with` statement
- Can use Python callback functions
  - `SNESSetFunction(), SNESSetJacobian()`
- Manages all memory (creation/destruction)
- Visualization with matplotlib

petsc4py Installation

- Configure PETSc using `-download-petsc4py`
  - Can also use `-download-mpi4py`
- Downloaded to `externalpackages/petsc4py-`*`version`*
  - Demo code is here
- Installed into PETSc lib directory
- Add `$PETSC_DIR/$PETSC_ARCH/lib` to **PYTHONPATH**

petsc4py Examples

- `externalpackages/petsc4py-1.1/demo/bratu2d/bratu2d.py`
  - Solves Bratu equation (SNES ex5) in 2D
  - Visualizes solution with matplotlib
- `src/ts/examples/tutorials/ex8.py`
  - Solves a 1D ODE for a diffusive process
  - Visualize solution using `-vec_view_draw`
  - Control timesteps with `-ts_max_steps`

ex8.py
Setup

```
import sys, petsc4py
petsc4py.init(sys.argv)
from petsc4py import PETSc
import math

# Create the grid
da = PETSc.DA().create([-9],comm=PETSc.COMM_WORLD)
f = da.createGlobalVector()
x = f.duplicate()
J = da.getMatrix(PETSc.Mat.Type.AIJ);

# Create the solver
ts = PETSc.TS().create(PETSc.COMM_WORLD)
ts.setProblemType(PETSc.TS.ProblemType.NONLINEAR)
ts.setType(ts.Type.GL)

# Define the problem
ode = MyODE(da)
ts.setIFunction(ode.function, f)
ts.setIJacobian(ode.jacobian, J)
```

ex8.py
Residual

```
class MyODE:
  def function(self, ts, t, x, xdot, f):
    mx = da.getSizes()[0];       hx = 1.0/mx
    (xs, xm) = da.getCorners(); xs = xs[0]; xm = xm[0]
    xx    = da.createLocalVector()
    xxdot = da.createLocalVector()
    da.globalToLocal(x, xx)
    da.globalToLocal(xdot, xxdot)
    dt = ts.getTimeStep()
    x0 = ts.getSolution()
    if xs == 0:      f[0]     = xx[0]/hx;            xs = 1;
    if xs+xm >= mx: f[mx-1] = xx[xm-(xs==1)]/hx; xm = xm-(xs==1);
    for i in range(xs, xs+xm-1):
      f[i] = xxdot[i-xs+1]
           + (2.0*xx[i-xs+1] - xx[i-xs] - xx[i-xs+2])/hx
           - hx*math.exp(xx[i-xs+1])
    f.assemble()
```

ex8.py
Solving

```
ts.setTimeStep(0.1)
ts.setDuration(10, 1.0)
ts.setFromOptions()
x.set(1.0)
ts.solve(x)
```

Outline

2 What's New in PETSc?
- Python Bindings
- Physics-Based Preconditioning
- PETSc-GPU
- FEM

MultiPhysics Paradigm

The **PCFieldSplit** interface

- extracts functions/operators corresponding to each physics
  - **VecScatter** and `MatGetSubmatrices()` for efficiency
- assemble functions/operators over all physics
  - Generalizes `LocalToGlobal()` mapping
- is composable with ANY PETSc solver and preconditioner
  - This can be done recursively

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FieldSplit provides the buildings blocks for multiphysics preconditioning.

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Notice that this works in exactly the same manner as

- multiple resolutions (MG, FMM, Wavelets)
- multiple domains (Domain Decomposition)
- multiple dimensions (ADI)

Preconditioning

Several varieties of preconditioners can be supported:

- Block Jacobi or Block Gauss-Siedel
- Schur complement
- Block ILU (approximate coupling and Schur complement)
- Dave May's implementation of Elman-Wathen type PCs

which only require actions of individual operator blocks

Notice also that we may have any combination of

- "canned" PCs (ILU, AMG)
- PCs needing special information (MG, FMM)
- custom PCs (physics-based preconditioning, Born approximation)

since we have access to an algebraic interface

Stokes Example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type

-fieldsplit_0_pc_type ml
-fieldsplit_0_ksp_type preonly
```

$$\begin{pmatrix} A & B \\ B^T & 0 \end{pmatrix}$$

Stokes Example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type additive

-fieldsplit_0_pc_type ml
-fieldsplit_0_ksp_type preonly

-fieldsplit_1_pc_type jacobi
-fieldsplit_1_ksp_type preonly
```

$$\begin{pmatrix} \hat{A} & 0 \\ 0 & I \end{pmatrix}$$

Stokes Example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type multiplicative

-fieldsplit_0_pc_type ml
-fieldsplit_0_ksp_type preonly

-fieldsplit_1_pc_type jacobi
-fieldsplit_1_ksp_type preonly
```

$$\begin{pmatrix} \hat{A} & B \\ 0 & I \end{pmatrix}$$

Stokes Example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type schur

-fieldsplit_0_pc_type ml
-fieldsplit_0_ksp_type preonly

-fieldsplit_1_pc_type none
-fieldsplit_1_ksp_type minres
-pc_fieldsplit_schur_factorization_type diag
```

$$\begin{pmatrix} \hat{A} & 0 \\ 0 & -\hat{S} \end{pmatrix}$$

Stokes Example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type schur

-fieldsplit_0_pc_type ml
-fieldsplit_0_ksp_type preonly

-fieldsplit_1_pc_type none
-fieldsplit_1_ksp_type minres
-pc_fieldsplit_schur_factorization_type lower
```

$$\begin{pmatrix} \hat{A} & 0 \\ B^T & \hat{S} \end{pmatrix}$$

Stokes Example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type schur

-fieldsplit_0_pc_type ml
-fieldsplit_0_ksp_type preonly

-fieldsplit_1_pc_type none
-fieldsplit_1_ksp_type minres
-pc_fieldsplit_schur_factorization_type upper
```

$$\begin{pmatrix} \hat{A} & B \\ 0 & \hat{S} \end{pmatrix}$$

Stokes Example

The common block preconditioners for Stokes require only options:

```
-pc_type fieldsplit
-pc_field_split_type schur
-pc_fieldsplit_schur_factorization_type full
```

$$\begin{pmatrix} I & 0 \\ B^T A^{-1} & I \end{pmatrix} \begin{pmatrix} \hat{A} & 0 \\ 0 & \hat{S} \end{pmatrix} \begin{pmatrix} I & A^{-1}B \\ 0 & I \end{pmatrix}$$

Outline

2 What's New in PETSc?
- Python Bindings
- Physics-Based Preconditioning
- PETSc-GPU
- FEM

Thrust

Thrust is a CUDA library of parallel algorithms

- Interface similar to C++ Standard Template Library
- Containers (`vector`, `list`, `map`) on both host and device
- Algorithms: `sort`, `reduce`, `scan`
- Freely available, and part of PETSc configure (`-with-thrust-dir`)

Cusp

Cusp is a CUDA library for
sparse linear algebra and graph computations

- Builds on data structures in Thrust
- Provides sparse matrices in several formats (CSR, Hybrid)
- Includes some preliminary preconditioners (Jacobi, SA-AMG)
- Freely available, and part of PETSc configure (`-with-cusp-dir`)

VECCUDA

Strategy: Define a new **Vec** implementation

- Uses Thrust for data storage and operations on GPU
- Supports full PETSc **Vec** interface
- Inherits PETSc scalar type
- Can be activated at runtime, `-vec_type cuda`
- PETSc provides memory coherence mechanism

Memory Coherence

PETSc Objects now hold a coherence flag

PETSC_CUDA_UNALLOCATED	No allocation on the GPU
PETSC_CUDA_GPU	Values on GPU are current
PETSC_CUDA_CPU	Values on CPU are current
PETSC_CUDA_BOTH	Values on both are current

Table: Flags used to indicate the memory state of a PETSc CUDA **Vec** object.

MATAIJCUDA

Also define new **Mat** implementations

- Uses Cusp for data storage and operations on GPU
- Supports full PETSc **Mat** interface, some ops on CPU
- Can be activated at runtime, `-mat_type aijcuda`
- Notice that parallel matvec necessitates off-GPU data transfer

Solvers

Solvers come for Free

- All linear algebra types work with solvers
- Entire solve can take place on the GPU
  - Only communicate scalars back to CPU
- GPU communication cost could be amortized over several solves
- Preconditioners are a problem
  - Cusp has a promising AMG

Installation

PETSc only needs

```
# Turn on CUDA
--with-cuda
# Specify the CUDA compiler
--with-cudac='nvcc -m64'
# Indicate the location of packages
#  --download-* will also work
--with-thrust-dir=/PETSc3/multicore/thrust
--with-cusp-dir=/PETSc3/multicore/cusp
# Can also use double precision
--with-precision=single
```

Example
Driven Cavity Velocity-Vorticity with Multigrid

```
ex19 -da_vec_type seqcuda
  -da_mat_type aijcuda -mat_no_inode # Setup types
  -da_grid_x 100 -da_grid_y 100      # Set grid size
  -pc_type none -dmmg_nlevels 1      # Setup solver
  -preload off -cuda_synchronize     # Setup run
  -log_summary
```

Outline

2 What's New in PETSc?
- Python Bindings
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Global and Local

Local (analytical)

- Discretization/Approximation
  - FEM integrals
  - FV fluxes
- Boundary conditions
- Largely dim dependent (e.g. quadrature)

Global (topological)

- Data management
  - Sections (local pieces)
  - Completions (assembly)
- Boundary definition
- Multiple meshes
  - Mesh hierarchies
- Largely dim independent (e.g. mesh traversal)

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- Multiple meshes
  - Mesh hierarchies
- Largely dim independent (e.g. mesh traversal)

Generic FEM

Can formulate assembly independent of

- spatial dimension
- element shape
- finite element (discretization)
- weak form (using FEniCS)

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  coords = mesh->restrict(coordinates, c);
  v0, J, invJ, detJ = computeGeometry(coords);
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
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  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  inputVec = mesh->restrict(U, c);
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    realCoords = J*refCoords[q] + v0;
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      elemVec[f] += basis[q,f]*rhsFunc(realCoords);
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Transform J */
      for(d = 0; d < dim; ++d)
        for(e = 0; e < dim; ++e)
          tDerReal[d] += invJ[e,d]*basisDer[q,f,e];
      for(g = 0; g < numBasisFuncs; ++g) {
        for(d = 0; d < dim; ++d)
          for(e = 0; e < dim; ++e)
            bDerReal[d] += invJ[e,d]*basisDer[q,g,e];
        /* Update element matrix */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Transform J */
      for(g = 0; g < numBasisFuncs; ++g) {
        for(d = 0; d < dim; ++d)
          elemMat[f,g] += tDerReal[d]*bDerReal[d];
        elemVec[f] += elemMat[f,g]*inputVec[g];
      }
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      elemVec[f] += basis[q,f]*lambda*exp(inputVec[f])
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  mesh->updateAdd(F, c, elemVec);
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
/* Aggregate updates */
```

Integration

```
cells = mesh->heightStratum(0);
for(c = cells->begin(); c != cells->end(); ++c) {
  /* Compute cell geometry */
  /* Retrieve values from input vector */
  for(q = 0; q < numQuadPoints; ++q) {
    /* Transform coordinates */
    for(f = 0; f < numBasisFuncs; ++f) {
      /* Constant term */
      /* Linear term */
      /* Nonlinear term */
      elemVec[f] *= weight[q]*detJ;
    }
  }
  /* Update output vector*/
}
Distribution<Mesh>::completeSection(mesh, F);
```

GPU

Preliminary system for FEM integration on a GPU

- High order is basically done by others
- Low order much more prevalent in applications
- Use PyCUDA and Mako to generate kernels
- Nearly 100GF on a GTX 285

Outline

1 What can PETSc do?

2 What's New in PETSc?

3 Conclusion

Why is PETSc cool?

PETSc gives you tools

to design and build

new Scientific Software

from simple pieces

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